1. For any numbers \( a, b \), let \( C_{a,b} \) be the curve in the domain \( x > 1 \) with the following parametrization: \( x = at + 1, \ y = t + b, \ z = at + b \). Find all vector fields \( V(x, y, z) \) in the domain \( x > 1 \) such that for any \( a, b \), the curve \( C_{a,b} \) is an integral curve of \( V \).

2. Solve the initial value problem in the domain \( x > 0 \): \( xy^3z_x + z^2z_y = -y^3z \), and \( z = y^2 \) on the hyperbola \( xy = 1 \).

3. Consider the differential equation \((z + 1)z_t + z_x = 0\) for \( t \geq 0 \).
   a) For the initial condition \( z(x, 0) = \frac{1}{x^2 + 1} \), find \( t_c \), the value of \( t \) when the shock develops. Sketch the graphs of \( z(x, 0) \) and \( z(x, t_c) \) as functions of \( x \). Point out the feature of the graph of \( z(x, t_c) \) that indicates the shock developing. The graphs should be sufficiently accurate.
   b) Find the minimum and maximum values of \( z(x, t) \) as \(-\infty < x < \infty , \ 0 \leq t < t_c \).
   c) Compute the integral \( \int_{-\infty}^{\infty} z(x, t_c) \, dx \).
   You do not need to find an analytic formula for the solution \( z(x, t) \).

4. Let \( \Omega \subset \mathbb{R}^2 \) be the domain \( x > 0, \ y > 0, \ x + y < 1 \).
Find all solutions of the boundary value problem
\[
\Delta u(x, y) = 0 \quad \text{for} \quad (x, y) \in \Omega, \\
u(x, y) = y^3 - 3x^2y \quad \text{for} \quad (x, y) \in \partial\Omega.
\]

5. Use separation of variables to solve the boundary value problem in the strip:
\[
\Delta u(x, y) = 3u \quad \text{for} \quad 0 < x < \pi/2, \ y > 0, \\
\frac{\partial u}{\partial n}(0, y) = 0, \quad u(\pi/2, y) = 0 \quad \text{for} \quad y > 0, \\
u(x, 0) = \cos^3 x, \quad \text{for} \quad 0 < x < \pi/2, \\
u(x, y) \text{ is bounded} \quad \text{as} \quad y \to \infty.
\]
6. Use D’Alembert’s method to solve the problem for the wave equation in the quadrant:

\[ 4u_{tt}(x, t) = u_{xx}(x, t), \quad x > 0, \quad t > 0, \]

\[ u(0, t) = \sin t, \quad t > 0, \]

\[ u(x, 0) = 0, \quad x > 0, \]

\[ u_t(x, 0) = 0, \quad x > 0. \]

Find the limit \( \lim_{x \to \infty} u(x, t) \) as a function of \( t \).

7. Let \( \Omega \) be a bounded normal domain in \( \mathbb{R}^3 \) and \( f \in C(\overline{\Omega} \times \mathbb{R}_{\geq 0}) \).

Prove that the initial boundary value problem

\[ u_t(x, t) = \Delta u(x, t) + f(x), \quad x \in \Omega, \]

\[ u(x, t) = 0, \quad x \in \partial \Omega, \]

\[ u(x, 0) = 0, \quad x \in \Omega. \]

has at most one solution in \( C^2(\overline{\Omega} \times \mathbb{R}_{\geq 0}) \).