Math 261 Sample Final Exam

Each question is worth 6 points.

1. Write down an integral for the arclength of the curve C given by the parametric equations $x = t^2$, $y = t^3$, $z = t + t^4$, for $0 \le t \le 2$, but DO NOT EVALUATE.

2. Give the equation of the tangent plane to the surface $2yz = x^2 + xy + xz^2$ at the point (1, 2, 1).

3. Find the directional derivative of $f(x, y) = \ln(x^2 + y^2)$ at (2, 1) in the direction of the vector $\mathbf{v} = \langle -1, 2 \rangle$.

4. Find $\dfrac{\partial f}{\partial v}$ if $f(x, y) = y\,x^2 + e^y$ and $x = 2u + 5v$, while $y = 3u - 7v$.

5. Find the maximum of the function $f(x, y) = x^2y$ subject to the constraint $x^2 + y^2 = 4$.

6. Calculate the iterated integral $\displaystyle\int_1^2 \int_0^{2x} \frac{y e^x}{x^2}\, dy\, dx$.

7. Set up the iterated integrals for the x and y coordinates of the center of mass of a lamina (thin plate) which occupies the region inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. DO NOT EVALUATE.

8. Find curl **F** if $\mathbf{F}(x, y, z) = e^{-x}\sin(y)\,\mathbf{i} + e^{-y}\sin(z)\,\mathbf{j} + e^{-z}\sin(x)\,\mathbf{k}$.

9. Set up and evaluate the integral $\displaystyle\iiint_E z^3\sqrt{x^2 + y^2 + z^2}\, dV$ over the solid region E which is the interior of the sphere of radius 5.

10. Calculate the integral $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y) = \dfrac{-y}{x^2 + y^2}\mathbf{i} + \dfrac{x}{x^2 + y^2}\mathbf{j}$ and C is the circle of radius a centered at the origin with counterclockwise orientation.

11. Set up an integral for the surface area of the part of the surface $z = x^2 + 2y$ that lies above the triangle with vertices (0, 0), (2, 0), (3, 2). Write as an iterated integral but DO NOT EVALUATE.

12. Use Green's Theorem to calculate the line integral $\oint_C x^2 y\, dx - x y^2\, dy$, where C is the circle of radius 2 with counterclockwise orientation.

13. Set up the integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x^2\,\mathbf{i} + x y\,\mathbf{j} + z\,\mathbf{k}$ and S is the part of the paraboloid $z = x^2 + y^2$ which lies below the plane $z = 1$, with upward pointing normal. Write as an iterated integral but DO NOT EVALUATE.

14. Use Stokes' Theorem to set up a line integral which is equal to the surface integral $\iint_S curl(\mathbf{G}) \cdot d\mathbf{S}$, where $\mathbf{G}$(x, y, z) = $y^2\,\mathbf{i} + z^2\mathbf{j} + x^2\mathbf{k}$ and S is the same surface as in Problem 13. Set up as in integral in one variable but DO NOT EVALUATE.

15. Set up and calculate the value of the line integral of the vector field $\mathbf{F}(x, y, z) = e^y\,\mathbf{i} + (x e^y + e^z)\,\mathbf{j} + y e^z\,\mathbf{k}$ along the line segment from (0, 2, 0) to (4, 0, 3).

BONUS Question. If $z = y + f\,(x^2 - y^2)$, where f is differentiable, show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = x\,.$$