Math 261 **Name_____**
Sample Test on Chs. 15-16
110 points

Credit will only be given if you show your work.

1. (10) Find the volume of the solid that lies above the paraboloid $z = x^2 + y^2$ and below the half-cone $z = \sqrt{x^2 + y^2}$.

2. (10) Find the center of mass of the solid in problem 1, if the density is a constant d.

3. (10) Find the mass of the ball given by the equation $x^2 + y^2 + z^2 \le a^2$ if the density at the point (x, y, z) is equal to the distance from the origin.

4. (10) Evaluate the integral $\int_0^1 \int_y^{2-y} e^{(x-y)/(x+y)}\, dx\, dy$ by means of the change of variables

$u = x - y$, $v = x + y$. (Hint: let R be the region of integration in terms of x-y coordinates. Find the image of this region in u-v coordinates under the change in variables.)

5. (15) **a**) Use the curl to show that the vector field

$$\mathbf{F}(x, y, z) = y z e^{xy}\,\mathbf{i} + (z x e^{xy} + z\cos(y))\,\mathbf{j} + (e^{xy} + \sin(y))\,\mathbf{k}$$

is conservative.

b) Find a potential function $f(x, y, z)$ for $\mathbf{F}(x, y, z)$ in a).

c) Use part b) to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the straight-line segment from the point $(1, \pi, 1)$ to the point $(2, 2\pi, 2)$.

6. (10) Set up and evaluate the integral for the surface area of the parametrized surface S given by the equations

$$x = u + v, \qquad y = u - v, \qquad z = 2u + 3v$$

for $0 \le u \le 1$ and $0 \le v \le 1$. Describe this surface.

7. (10) Evaluate the line integral $\oint_C y\,dx + (x^2 + y)\,dy$, where C is the boundary of the rectangle whose vertices are (2, 1), (5, 1), (5, 3), and (2, 3), with counter-clockwise orientation.

8. (10) A particle starts at the point (-2, 0), moves along the x-axis to the point (2, 0), and then along the semi-circle $y = \sqrt{4 - x^2}$ to the starting point. Use Green's theorem to find the work done on this particle by the force field $\mathbf{F}(x, y) = \langle x, x^3 + 3xy^2 \rangle$.

9. (10) What is the surface area of the part of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices (0, 0), (0, 1), and (2, 1)? (Hint: choose the order in which you do the integration carefully.)

10. (15) Express the iterated volume integral $V = \int_0^4 \int_0^{4-z} \int_{-\sqrt{y}}^{\sqrt{y}} 1\,dx\,dy\,dz$ in the following three ways as an iterated integral. Sketch the solid whose volume is given by this integral.

a) Using the order of integration *dx dz dy*.

b) Using *dz dx dy*.

c) Using *dy dz dx*.