Differential Equations

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Objective

In this project we investigate the solutions of some differential equations.

Narrative

In this project we introduce the command `dsolve`:

|	
`dsolve('Dx=-2*x')`	solves the first-order equation $x' = -2x$.
`dsolve('Dx=-2*x','x(0)=3')`	solves the first-order equation $x' = -2x, x(0) = 3$.
`dsolve('D2x+4*Dx+4*x=0','x(0)=1','Dx(0)=0')`	solves the second-order equation $x'' + 4x' + 4x = 0$, $x(0) = 1, x'(0) = 0$.

Tasks

1. Type the commands in the left-hand column below into MATLAB. These commands solve some simple first order differential equations. (Is MATLAB's output correct? You should be able to solve these equations easily!)

   ```
   >> % Your name, today's date
   >> % Differential Equations
   >> clear all, close all
   >> % Task 1
   >> x = dsolve('Dx=-2*x')
   ```
   Find the general solution to the first-order equation $x' = -2x$.

   ```
   >> x = dsolve('Dx=-2*x','x(0)=3')
   ```
   Find the specific solution to the first-order equation $x' = -2x, x(0) = 3$.

   ```
   >> figure(1), ezplot(x,[-1.0,1.0])
   ```
   Graph the last solution in the Figure 1 window.

2. Continue by typing the commands below into MATLAB. The function $x_1 = x_1(t)$ is the solution to the differential equation

$$x'' + 4x = 0, x(0) = 1, x'(0) = 0.$$

This is the equation governing the motion of a spring-mass system consisting of a spring whose spring constant $k = 4$, whose mass is a unit mass $m = 1$, with no damping, and which is released after being stretched $x(0) = 1$ unit out of equilibrium (a simple harmonic motion). To graph x_1 so that it can be properly interpreted in this context, we have used the command `set(gca,'ydir','reverse')` to reverse the y-direction in the Figure 2 window.
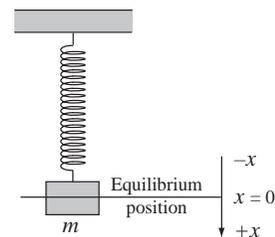
The function $x_2 = x_2(t)$ is the solution to the differential equation

$$x'' + x' + 4x = 0, x(0) = 1, x'(0) = 0.$$

This is the equation governing the motion of the above spring-mass system, but now with a small amount of damping (underdamping). (The purpose of the `set(gca,'ylim',[-0.2,1.0])` command is to restrict the range over which x_2 is graphed.) The function $x_3 = x_3(t)$ is the solution to the differential equation

$$x'' + 5x' + 4x = 0, x(0) = 1, x'(0) = 0.$$

This is the equation governing the motion of the above spring-mass system, but now with a large amount of damping (overdamping). And the function $x_4 = x_4(t)$ is the solution to the differential equation

$$x'' + 4x' + 4x = 0, x(0) = 1, x'(0) = 0.$$

This is the equation governing the motion of the above spring-mass system, but now with a critical amount of damping. (Any less damping results in underdamping, and any more damping results in overdamping.)

Note: Pay careful attention to the shape of the curves drawn by each command in this task. At the end of this task the Figure 2 window will contain the graphs of 4 functions, and at the end of this project you will be asked to highlight and label each graph by hand. To simplify labeling each graph, you might, in fact, find it useful to make a preliminary hard copy of the Figure 2 window after drawing each graph.

```
>> % Task 2
>> x1 = dsolve('D2x+4*x=0','x(0)=1.0','Dx(0)=0.0')
>> figure(2)
>> hold on, ezplot(x1,[0,2*pi], set(gca,'ydir','reverse'), hold off
>> x2 = dsolve('D2x+Dx+4*x=0','x(0)=1.0','Dx(0)=0.0')
>> hold on, ezplot(x2,[0,2*pi]), set(gca,'ylim',[-0.2,1.0]), hold off
>> x3 = dsolve('D2x+5*Dx+4*x=0','x(0)=1.0','Dx(0)=0.0')
>> hold on, ezplot(x3,[0,2*pi]), set(gca,'ylim',[-0.2,1.0]), hold off
>> x4 = dsolve('D2x+4*Dx+4*x=0','x(0)=1.0','Dx(0)=0.0')
>> hold on, ezplot(x4,[0,2*pi]), set(gca,'ylim',[-0.2,1.0]), hold off
```

3. Continue by typing the commands below into MATLAB. The purpose of these commands is to illustrate the effect of having a forcing function $f(t)$ in a differential equation of the form

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = f(t).$$

We begin by redrawing the graph of x_4 from Task 2 in Figure 3, and then drawing the graph of the solution $x_5 = x_5(t)$ to the differential equation $x'' + 4x' + 4x = 2\sin 4t, x(0) = 1, x'(0) = 0$.

```
>> % Task 3
>> figure(3)
>> hold on, ezplot(x4,[0,2*pi]), set(gca,'ydir','reverse'), hold off
>> x5 = dsolve('D2x+4*Dx+4*x=2*sin(4*t)','x(0)=1.0','Dx(0)=0.0')
>> hold on, ezplot(x5,[0,2*pi]), set(gca,'ylim',[-0.2,1.0]), hold off
```

At this time, make a hard copy of MATLAB's Command Window and hard copies of its Figure 1–3 windows. If you made any typing errors, neatly draw a line through them and any resulting MATLAB output, by hand. Then:

3. On the hard copy of your Figure 1 window, draw and label the t- and x-coordinate axes (be careful!).

4. On the hard copy of your Figure 2 window, draw and label the t- and x-coordinate axes (be careful!). Then highlight in 4 different colors the graphs of x_1, x_2, x_3, and x_4, and label the graph of x_1 by "simple harmonic motion", the graph of x_2 by "underdamped motion", the graph of x_3 by "overdamped motion", and the graph of x_4 by "critically damped motion". (Only work on the final hard copy of the Figure 2 window: any preliminary hard copies of the Figure 2 window are for your reference only, and need not be supplemented by hand or turned in.)

5. On the hard copy of your Figure 3 window, draw and label the t- and x-coordinate axes (be careful!). Then highlight in 2 different colors the graphs of x_4 and x_5, and label the graph of x_4 by "unforced motion" and the graph of x_5 by "forced motion".

   Your lab report for this project will be your hard copies of the Command Window and the Figure 1, 2, and 3 windows.