

Radioactive Decay

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Objective

To study the decay of a radioactive material.

Narrative

Certain materials decay (they lose electrons, and hence mass) as time passes; these materials are known as *radioactive materials*, and the process of losing mass over time is known as *radioactive decay*. The number of grams of a radioactive material present at time t may be described by a function of the form

$$m(t) = m_0 a^t$$

where m_0 is the mass at time $t = 0$ and $a \in (0, 1)$ is a constant.

One example of such a function is

$$m(t) = m_0 \left(\frac{1}{2}\right)^t$$

Observe that $m(0) = m_0$, so m_0 is the mass of the material at time $t = 0$. And $m(1) = m_0/2$ so that when $t = 1$ there is half as much of the material left as there was at time $t = 0$. In general, there is half as much of the material left at time $t + 1$ as there was at time t :

$$m(t + 1) = m_0 \left(\frac{1}{2}\right)^{t+1} = m_0 \left(\frac{1}{2}\right)^t \frac{1}{2} = \frac{1}{2}m(t).$$

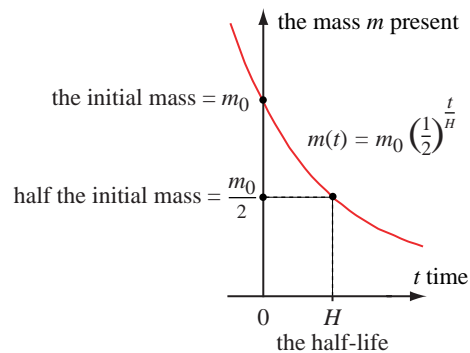
A (more general) example of such a function is

$$m(t) = m_0 \left(\left(\frac{1}{2}\right)^{1/H}\right)^t = m_0 \left(\frac{1}{2}\right)^{t/H}$$

where H is a constant known as the *half-life* of the material. Again, $m(0) = m_0$ so m_0 is the mass of the material present at time $t = 0$. But now it is when $t = H$ that there is half as much of the material left as there was when $t = 0$, for

$$m(H) = m_0 \left(\frac{1}{2}\right)^{H/H} = \frac{1}{2}m_0.$$

This is illustrated in the figure to the right.



Tasks

1. Type the command lines below into MATLAB. These commands produce a graph of $m(t) = 10e^{-0.1254t}$.

```
>> % Your name, today's date
>> % Radioactive Decay
>> clear all, close all
>> m = inline('10*exp(-0.1254*t)')
>> ezplot(m, [0.0,10.0,0.0,10.0])
```

At this time, make a hard copies of MATLAB's Command Window and its Figure 1 window. If you made any typing errors, neatly draw a line through them and any resulting MATLAB output, by hand. Then:

2. On the graphic you created in Task 1:

- (a) estimate the half-life of the material whose mass is given by $m(t) = 10e^{-0.1254t}$, by drawing lines and labeling intercepts as in the figure above, by hand,
- (b) write a statement of the form, “The half-life is ____.” by hand, filling in the blank with your estimate H of the half-life, and
- (c) check your estimate H of the half-life by computing $m(H)$ to 6 decimal places of accuracy by hand, and writing a statement of the form “ $m(\text{____}) = \text{____}$.” filling in the blanks with the appropriate values of H and $m(H)$.

Your lab report will be a hard copy of your typed input and MATLAB’s responses (both text and hand-labeled graphics).

Comments

To find the half-life H of a radioactive substance given that m_0 grams are initially present and that $m(t) = m_0e^{-kt}$ is present at time t , we need to find the time H at which

$$m(H) = m_0e^{-kH} = \frac{1}{2}m_0.$$

We solve this equation by dividing by m_0 :

$$e^{-kH} = \frac{1}{2}$$

taking the ln of each side:

$$-kH = \ln \frac{1}{2}$$

and dividing by $-k$:

$$H = \frac{\ln \frac{1}{2}}{-k} = \frac{-\ln 2}{-k} = \frac{\ln 2}{k}.$$

Can you see how you could determine the half-life H of a radioactive substance given that m_1 grams are present at time $t = t_1$ and that m_2 grams are present at time $t = t_2$?