

MATH 22100

Sample FINAL EXAM Problems

SHOW YOUR WORK, PLEASE

Problem 1 Find:

a. $\lim_{x \rightarrow \infty} \frac{x^2 - 3}{5 - 2x^2}$

b. $\lim_{x \rightarrow 4} \frac{x^2 - 3}{5 - 2x^2}$

c. $\lim_{x \rightarrow 2} \frac{x^2 - 5x + 6}{x^2 + x - 6}$

Problem 2 Use the definition (in other words, use the delta-process or the Four-step process) to find y' if $y = x^2 + 4x - 2$

Problem 3 Differentiate by using differentiation rules (not the delta process):

$$a. f(x) = \frac{x^2 + 1}{3x - 4}$$

$$b. f(x) = x^2 \ln 3x$$

$$c. f(x) = \text{Arctan}(4x^3 + 5)$$

$$d. f(x) = \sin^5 5x$$

Problem 4 Find the equations of the line TANGENT and NORMAL to the curve $y = x^4 + 2x - 2$ at the point $(1,3)$.

Problem 5 If $x^2y^3 + y = 2x + e^y$, find y' by differentiating implicitly:

Problem 6 A playing field is to be built in the shape of a rectangle plus a semicircular area at each end. A 440-yd race track is to form the perimeter of the field. Find the dimensions of the field if the rectangular part is to have as large an area as possible.

Problem 7 Find dy if

$$y = \frac{1}{\sqrt[3]{x}}$$

Problem 8 a. Find the intervals on which the function $y = 2x^3 - 3x^2 + 2$ is increasing and decreasing

b. Find the extreme values of the given function

c. Find the intervals on which the graph is concave up and is concave down

Problem 9 Sketch the graph of the function $f(x) = \frac{x}{x-1}$. This function has the following derivatives:

$f'(x) = -\frac{1}{(x-1)^2}$ and $f''(x) = \frac{2}{(x-1)^3}$. DO NOT recalculate the derivatives; just USE them!

Problem 10 Integrate

a. $\int (x^3 - 2x^{-4})dx$

b. $\int_0^2 x\sqrt{x^2 + 5}dx$

Problem 11 Find the area bounded by the indicated curves.

$$y = x^2, y = -x$$

Problem 12 The region R is bounded by the curves $y = x^3 - 1$, x -axis, $x = 1$ and $x = 3$.

a. Set up an integral for the volume generated by rotating this region about the x -axis. Use disks.

b. Set up an integral for the volume generated by rotating this region about the y -axis. Use shells.