1. Label the ten vertices in the following plane diagram with digits 0 through 9, using each digit exactly once, so that
   (i) the values at the vertices of each of the three triangles add to a constant sum $T$; and
   (ii) the values at the vertices of each of the three pentagons add to another constant sum $P$.
   You may use the template available at
   https://math.iupui.edu/math/highschoolmathcontest

   Below are two attempts: one satisfies $T$, but not $P$; and the other satisfies $P$, but not $T$. Can you produce a labelling that satisfies both $T$ and $P$?

   (a) A $T$-constant labelling ($T=13$)  
   (b) a $P$-constant labelling ($P=26$)
2. Let P be a regular n-gon inscribed in the unit circle and let Q be a regular 2n-gon inscribed in the unit circle. Express the length of a side of Q in terms of the length of a side of P.

3. Keep rolling a fair, six-faced die, and keep adding the outcomes until the running total exceeds 1000. When you stop, the running total must be 1001, 1002, ..., or 1006; but with what probabilities? (Approximate solutions are welcome if you cannot find an exact one.)

4. Let \( x_1, ..., x_n \) be real numbers in the interval \([-a, b]\), where both a and b are positive. Suppose that \( x_1 + ... + x_n = 0 \). Prove that \( x_1^2 + x_2^2 + ... + x_n^2 \leq nab \).

**Team Problems**

1. Consider a polynomial \( p_c(x) = x^2 + c \). The variable \( c \) is thought of as a “parameter” which will be fixed at various chosen values throughout the problem. Let \( p_c^{on}(x) \) denote the composition of \( p_c(x) \) with itself \( n \) times, for example \( p_c^{o2}(x) = p_c(p_c(p_c(x))) \). A point \( x \) is called periodic for \( p_c \) if there exists a natural number \( n \) such that \( p_c^{on}(x) = x \). The smallest such \( n \) is called the “period” of \( x \).
   a) How many integer values of \( x \) are periodic points for \( p_{-1}(x) = x^2 - 1 \)?
   b) Can you find an integer value of the parameter \( c \) such that \( p_c(x) \) has an integer periodic point \( x \) whose period is exactly 3?
   c) Prove for any integer parameter \( c \) the polynomial \( p_c(x) \) has at most finitely many integer points \( x \) that are periodic.
   d) Is there a uniform bound \( M \) such that for all parameters \( c \) the polynomial \( p_c(x) \) has at most \( M \) periodic points?

2. (a) \( 3^3 + 4^3 + 5^3 = 6^3 \). Show how to cut a 6x6x6 cube into as few pieces as possible (composed of sets of contiguously connected 1x1x1 cubes) that can be reassembled into a 3x3x3 cube, a 4x4x4 cube, and a 5x5x5 cube.
   (b) \( 10^3 + 9^3 = 12^3 + 1^3 \). Same problem cutting a 12x12x12 cube into as few pieces as possible which, when combined with the 1x1x1 cube, can be reassembled into a 10x10x10 cube and a 9x9x9 cube.