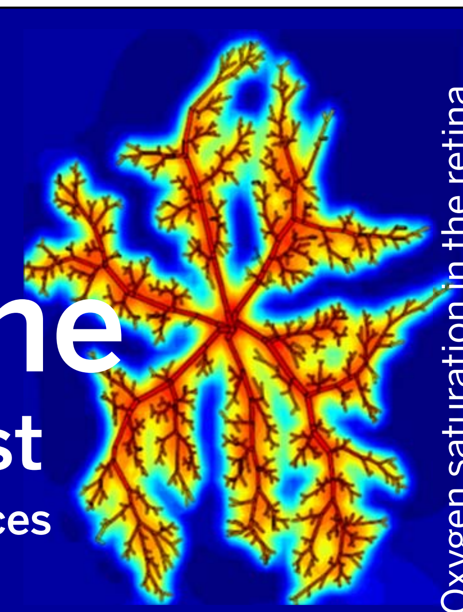
IUPUI

Math and Medicine

2019 High School Math Contest

Presented by the Department of Mathematical Sciences

https://math.iupui.edu/math/highschoolmathcontest



Contest opens!
January 19th

Submission Date:
March 8th

Awards Ceremony:
April 12th

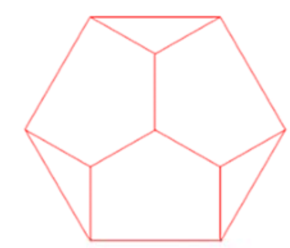
Event Chairs:
Dr. Roland Roeder
Dr. Maxim Yattselev

Keynote Speaker:
Dr. Julia Arciero

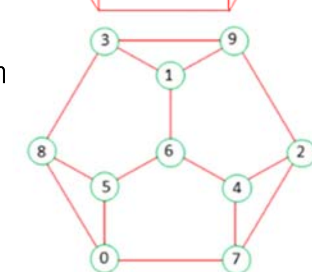
One 1st place prize • $300 and a full 4-year tuition scholarship
Five 2nd place prizes • $150 each and a $2,500/yr scholarship
Ten 3rd place prizes • $100 each and a $2,500/yr scholarship
Honorable mentions will receive a gift
Scholarship details posted on our website

Solve 1! Solve some! Solve all!
Be sure to tell us your reasoning and cite sources.
Do it by March 8th and follow all of the rules on the website for submission!
https://math.iupui.edu/math/highschoolmathcontest

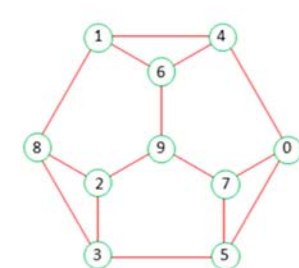
Individual Problems



1. Label the ten vertices in the plane diagram to the right with digits 0 through 9, using each digit exactly once, so that (i) the values at the vertices of each of the three triangles add to a constant sum T; and (ii) the values at the vertices of each of the three pentagons add to another constant sum P. Two incorrect attempts are shown: one satisfies T, but not P; and the other satisfies P, but not T. Can you produce a labelling that satisfies both T and P?
   You may use the template available at https://math.iupui.edu/math/highschoolmathcontest
2. Let P be a regular n-gon inscribed in the unit circle and let Q be a regular 2n-gon inscribed in the unit circle. Express the length of a side of Q in terms of the length of a side of P.
3. Keep rolling a fair, six-faced die, and keep adding the outcomes until the running total exceeds 1000. When you stop, the running total must be 1001, 1002, …, or 1006; but with what probabilities? (Approximate solutions are welcome if you cannot find an exact one.)
4. Let $x_1,\ldots,x_n$ be real numbers in the interval $[-a,b]$, where both a and b are positive. Suppose that $x_1+\ldots+x_n = 0$. Prove that $x_1^2+x_2^2+\ldots+x_n^2 \le nab$.
5. Write 500 to 700 words (complete with references) on an application of mathematics to medicine.



(a) A T-constant labelling (T=13)



(b) a P-constant labelling (P=26)

Team Problems

1. Consider a polynomial $p_c(x) = x^2 + c$. The variable c is thought of as a "parameter" which will be fixed at various chosen values throughout the problem. Let $p_c^{\circ n}(x)$ denote the composition of $p_c(x)$ with itself n times, for example $p_c^{\circ 3}(x) = p_c(p_c(p_c(x)))$. A point x is called periodic for p_c if there exists a natural number n such that $p_c^{\circ n}(x) = x$. The smallest such n is called the "period" of x.
   a) How many integer values of x are periodic points for $p_{-1}(x) = x^2-1$?
   b) Can you find an integer value of the parameter c such that $p_c(x)$ has an integer periodic point x whose period is exactly 3?
   c) Prove that for any integer parameter c the polynomial $p_c(x)$ has at most finitely many integer points x that are periodic.
   d) Is there a uniform bound M such that for all integer parameters c the polynomial $p_c(x)$ has at most M integer periodic points?
2. (a) $3^3 + 4^3 + 5^3 = 6^3$. Show how to cut a 6x6x6 cube into as few pieces as possible (composed of sets of contiguously connected 1x1x1 cubes) that can be reassembled into a 3x3x3 cube, a 4x4x4 cube, and a 5x5x5 cube.
   (b) $10^3 + 9^3 = 12^3 + 1^3$. Same problem cutting a 12x12x12 cube into as few pieces as possible which, when combined with the 1x1x1 cube, can be reassembled into a 10x10x10 cube and a 9x9x9 cube.

School Award:	Spirit Award:
2018 • Fishers High School	2018 • Avon High School
2017 • Carmel High School	2017 • Avon High School
2016 • Fishers High School	2016 • Avon High School

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Mu Alpha Theta National Mathematics Honor Society for High School Students

You're invited!

IUPUI cordially invites all participants, parents, and educators to the awards ceremony on **April 12, 2019**. Dr. Julia Arciero, Professor in the IUPUI Department of Mathematical Sciences, is giving the keynote speech. Winners will be announced and refreshments will be served. Details on our website and in separate e-mail to follow.

IUPUI High School Math Contest, Department of Mathematical Sciences, 317.274.6918 or mcontest@iupui.edu