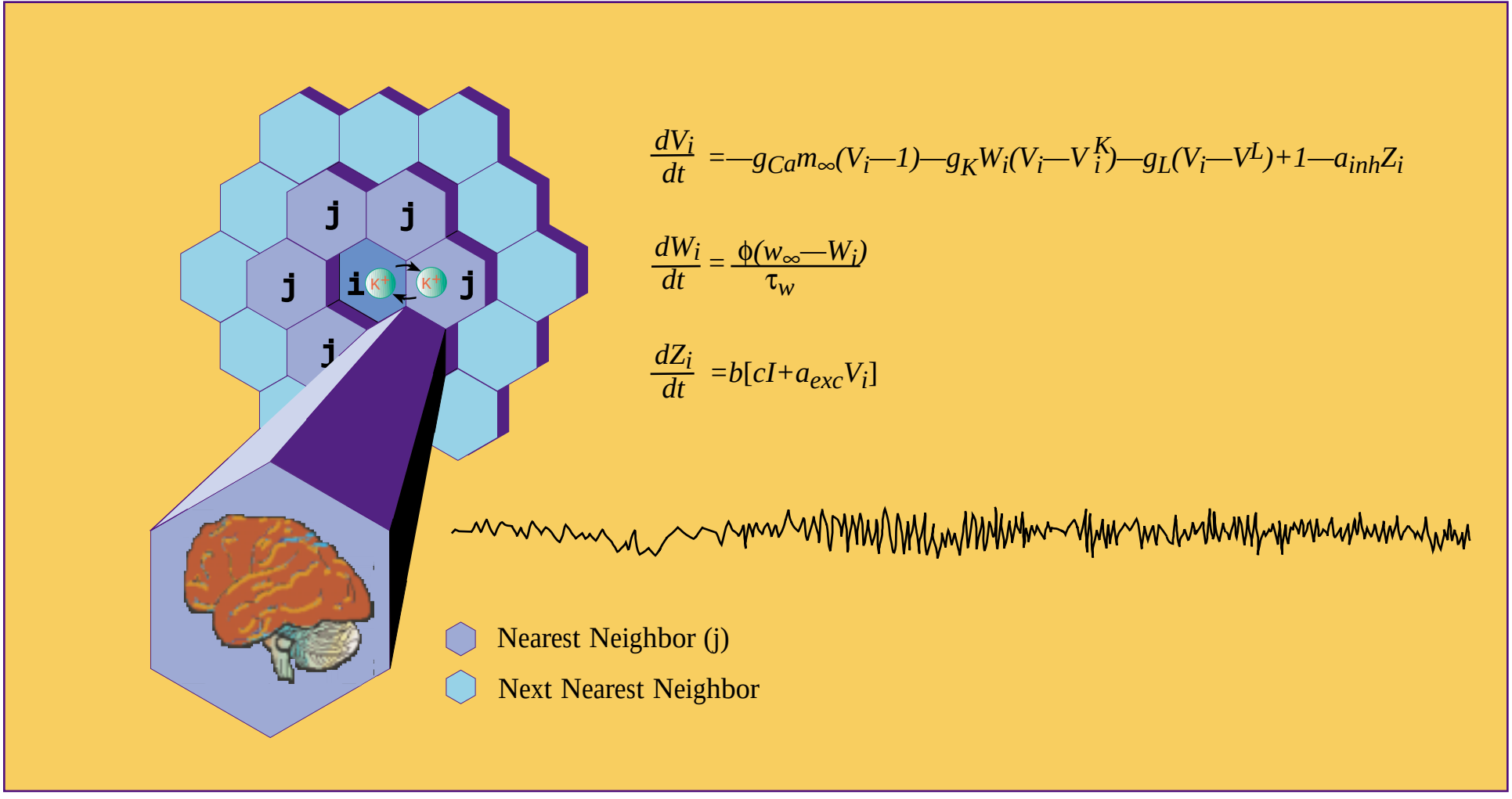

MATHEMATICS IN THE STUDY OF EPILEPSY**.** An epileptic seizure takes place when one small abnormal area in the brain, the seizure focus, recruits surrounding normal brain regions. The process can be modeled mathematically by the system of three differential equations. They describe the time evolution of the seizure as an abnormal hexagonal area, *i*, affects its nearest neighbors, *j*, through simple potassium diffusion. An electroencephalogram (EEG) of an actual seizure is shown below the equations.

Poster graphics courtesy of Robert Worth, M.D., Dept. of Neurological Surgery, and Professor Raima Larter, Ph.D., and Brent Speelman, Ph.D., Dept. of Chemistry.

IUPUI / Roche Diagnostics 2001 High School Mathematics Contest

Presented by
The IUPUI Department of Mathematical Sciences and Roche Diagnostics Corporation

PRIZES:

First prize — $500 15 second prizes — $200 each
Honorable mentions — popular book or video on mathematics.

Five of the $200 prizes will reward especially elegant submissions for each individual problem. All entrants will receive certificates honoring their participation. The school whose students have the best overall performance will be awarded a traveling trophy. Schools awarded the trophy in the past:

Brebeuf Jesuit Preparatory School, 1998
Roncalli High School, 1999
Carmel High School, 2000

CEREMONY:

Prizewinners and honorable mentions will be invited to an awards ceremony at IUPUI on Friday, May 11, 2001, from 4:00 to 6:30 PM. Parents and teachers will also be invited. The program will feature the awards presentation, catered refreshments and a talk on applications of mathematics in medicine.

ENTRIES:

Submit your entry by Friday, April 13, 2001. You may obtain a copy of the questions, the instructions for mailing your entry and the cover sheet from your math teacher (or by contacting the IUPUI Math Department). Solve the questions, giving your reasoning, not just the answer. Entries will be judged by professors in the IUPUI Department of Mathematical Sciences. Judging will be based on elegance of solution as well as correctness.

CONTACT INFORMATION: www.math.iupui.edu/contest

IUPUI/Roche Diagnostics Mathematics Contest
Department of Mathematical Sciences
402 North Blackford Street
Indianapolis, IN 46202-3216
(317) 274-MATH contest@math.iupui.edu

QUESTIONS:

1. In a town, 2/3 of the men are married to 3/4 of the women. What fraction of the adult population is married?

2. The numbers 1 through 100 are assigned sequentially to the blocks of a 10 by 10 square as indicated.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
91	92	93	94	95	96	97	98	99	100

   Then minus signs are attached to half of them, in such a way that in each row and in each column, half of the numbers are positive and half are negative. Show that the sum of the resulting 100 numbers is independent of how the minus signs are distributed and find this sum.

3. You are given 2 marbles and the task of determining the highest story of a 100 story building from which you can drop a marble without breaking it. Find a scheme that is guaranteed to give the answer in the fewest number of drops.

4. Let $A_1A_2A_3A_4A_5A_6A_7$ be a regular heptagon. Prove that

$$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}.$$

5. Write an essay of 500 to 700 words (complete with bibliography) on an application of mathematics to medicine.

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Roche

SPECIAL THANKS TO:

Professor Marvin L. Bittinger Professor Emeritus Conrad Crown Professor Jeffrey X. Watt

Answers to the 2001 IUPUI/Roche Diagnosics Mathematics Contest

1. In a town, 2/3 of the men are married to 3/4 of the women. What proportion of the population is married?

Solution. Let m be the total number of men and w be the total number of women. Given $\frac{2}{3}m = \frac{3}{4}w$, $m = \frac{9}{8}w$. So the total number of adults in the town is $m+w = \frac{9}{8}w+w = \frac{17}{8}w$. The number of married people is $\frac{2}{3}m + \frac{3}{4}w = \frac{3}{4}w + \frac{3}{4}w = \frac{3}{2}w$. Thus the fraction of married people is $\frac{3}{2}w/\frac{17}{8}w = \frac{12}{17}$.

2. The numbers 1 through 100 are assigned sequentially to the blocks of a 10 by 10 square. Then minus signs are attached to half of them, in such a way that in each row and in each column, half of the numbers are positive and half are negative. Show that the sum of the resulting 100 numbers is independent of how the minus signs are distributed, and find this sum.

Solution. Think of each row as a sum of the vector $\langle 1, 2, \ldots, 10\rangle$ and a vector of constants $\langle c_i, c_i, \ldots, c_i\rangle$. The constant depends on the row: $c_1 = 0, c_2 = 10, c_3 = 20, \ldots, c_{10} = 90$. When adding by rows first, since half of the signs in each row are plus and half are minus, all of the constants will cancel off. So without loss of generality, each row consists of $1, 2, \ldots, 10$, with signs. Then when adding by columns, since half the signs in each column are plus and half are minus, the columns sums are zero. So we obtain a sum of zero for any original sign distribution.

3. You are given 2 marbles and the task of determining the highest story of a 100 story building from which you can drop a marble without breaking it. Find a scheme that is guaranteed to give the answer in the fewest number of drops.

Solution. The key observation to make is, if you drop the first marble from a certain floor, say floor 13, then 13 is the best number you can hope to find for the fewest number of drops. This is because if it breaks from floor 13, you will have to drop the other marble

from floor 1, floor 2, all the way up to floor 12 for a total number of 13 drops. On the other hand if it does not break from floor 13, and you want to try to hold the number of drops to 13, since you have already used one drop you can go up at most 12 more floors for the next drop. If it breaks, you have a total of 13; if not, go up 11 more floors. By the time you have made 13 drops that do not break you are at floor $13 + 12 + 11 + \ldots + 1 = 91$. So you could not reach the top. Starting at floor 14, the same reasoning shows we can succeed in 14 or fewer drops. In summary, we are looking for the smallest value of n such that

$$n + (n-1) + (n-2) + \ldots + 1 = \frac{n(n+1)}{2} \geq 100$$

4. Let $A_1A_2A_3A_4A_5A_6A_7$ be a regular heptagon. Prove that

$$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}.$$

Solution (Michael Chang, Hamilton Southeastern). Inscribe the heptagon inside a circle. The vertices A_1, A_2, A_3, A_5 then define an inscribed quadrilateral with diagonals of lengths A_2A_5 and A_1A_3. By Ptolemy's Theorem,

$$A_1A_3 \times A_2A_5 = A_1A_2 \times A_3A_5 + A_2A_3 \times A_1A_5$$

As the heptagon is regular, $A_2A_5 = A_1A_4 = A_1A_5$, $A_1A_2 = A_2A_3$ and $A_1A_3 = A_3A_5$. By substitution we get

$$A_1A_3 \times A_1A_4 = A_1A_2 \times A_1A_3 + A_1A_2 \times A_1A_4$$

Now divide by $A_1A_2 \times A_1A_3 \times A_1A_4$.

Other solutions are possible using trigonometry.

2001 High School Math Contest Awards

First Prize Winner

- Jonathan Steven Landy, Warren Central High School. Teacher: Ms. Julia Oblon.

Second Prize Winners

Problem 1:

- Sandy Ottensmann, Brebeuf Jesuit Preparatory School. Teacher: Ms. Joan Rocap.

Problem 2:

- Matthew Adam Fisher, Brebeuf Jesuit Preparatory School. Teacher: Mrs. Sandra Laycock.

Problem 3:

- Irene Yuan Sun, Ben Davis High School. Teacher: Mr. Richard Elmore.

Problem 4:

- Michael Chun Chang, Hamilton Southeastern High School. Teacher: Ms. Susan Wong.

Problem 5:

- Kathleen Wolter, Avon High School. Teacher: Mr. Tony Record.

Overall:

- Aileen Chen, Carmel High School. Teacher: Mrs. Nancy Schulenburg.
- Ivan Y. Dremov, North Central High School. Teacher: Mrs. Jan Wendt.
- Karen Lai, Carmel High School. Teacher: Mrs. Nancy Schulenburg.
- Christopher Merryman, Carmel High School. Teacher: Mrs. Laura Diamente.
- Patrick Joseph Mihelich, Park Tudor. Teacher: Mrs. Joanne Black.
- Caitlin Elizabeth Pauckner, Brebeuf Jesuit Preparatory School. Teacher: Ms. Joan Rocap.
- Michael Jonathan Star, Brebeuf Jesuit Preparatory School. Teacher: Mrs. Sandra Laycock.
- Feng Tu, Hamilton Southeastern High School. Teacher: Ms. Susan Wong.
- Matt Willsey, Roncalli High School. Teacher: Mrs. Bonnie Ramey.
- Jerry Wu, Carmel High School. Teacher: Mrs. Kathie Freed.

Honorable Mention Winners

- David Bauman, Roncalli High School. Teacher: Mrs. Bonnie Ramey.
- Barbara Brill, Roncalli High School. Teacher: Sr. Anne Frederick.

- Erika Hollins Dantzig, Brebeuf Jesuit Preparatory School. Teacher: Mr. Tim Kelaghan.
- Scott Adam Dial, Ben Davis High School. Teacher: Mr. Wallace Mack.
- John Durham, Warren Central High School. Teacher: Mr. John Greenlee.
- J. Alex Emerson, Hamilton Southeastern High School. Teacher: Ms. Susan Wong.
- Greg Hasty, Warren Central High School. Teacher: Mrs. Janis Gaerte.
- Amy L. Hoffman, Carmel High School. Teacher: Mrs. Nancy Schulenburg.
- Nicholas Flynn Hoover, Brebeuf Jesuit Preparatory School. Teacher: Ms. Joan Rocap.
- Henry Jung, Ben Davis High School. Teacher: Mr. Richard Elmore.
- Steven Linville, Franklin Community High School. Teacher: Mr. Jason Boone.
- Gregory Ryan Martens, Brebeuf Jesuit Preparatory School. Teacher: Mr. Tim Kelaghan.
- Bhumi Dinesh Rajkotia, North Central High School. Teacher: Mr. Paul Brown.
- Lisa Ann Schaus, Brebeuf Jesuit Preparatory School. Teacher: Mrs. Sandra Laycock.
- Ryan W. Tobin, Warren Central High School. Teacher: Mrs. Janis Gaerte.
- Alexander Hilmes Toumey, Brebeuf Jesuit Preparatory School. Teacher: Mrs. Sandra Laycock.
- Chong Yan, Carmel High School. Teacher: Mrs. Kathie Freed.