

April 30, 2011

MATH 166

FINAL EXAM SPRING 2011 (100 points)

SHOW YOUR WORK PLEASE

Problem 1 (14 points) Find an equation of the tangent line to the curve $y = e^{x^3} \arcsin(x^2)$ when $x = 0$.

ANSWER:



Problem 2 (7 points) Find the absolute maximum and minimum values for the function $f(x) = \ln^2 x$, on the interval $[\frac{1}{e}, e^2]$.

ANSWER:



Problem 3 (21 points) Evaluate

(a) $\int x \arctan x^2 dx.$

ANSWER:


(b) $\int \cos^2 2y \sin^2 2y dy.$

ANSWER:

(CONTINUATION PROBLEM 3)

(c)
$$\int \frac{e^{4x}}{e^{8x} + 3e^{4x} + 2} dx.$$

ANSWER:



Problem 4 (8 points) Determine if the following *improper* integral is convergent or divergent. If it is convergent, find its value:

$$\int_0^1 \frac{x-3}{2x-3} dx.$$

ANSWER:



Problem 5 (8 points) A bacteria population grows in time at a rate that is proportional with the present population. The double life time for this strain of bacteria is 4 hours.

Find out how long a lab rat has been infected with this bacteria if the current size of the bacterial population in its body is 450 percent of the original injected sample.

ANSWER:



Problem 6 (7 points) Evaluate the following limit:

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{\arcsin x}.$$

ANSWER:

Problem 7 (7 points) On the following curve, find all the points where the tangent line is horizontal or vertical:

$$x = \sin(t), \quad y = \cos(3t), \quad t \text{ in } [0, 2\pi]$$

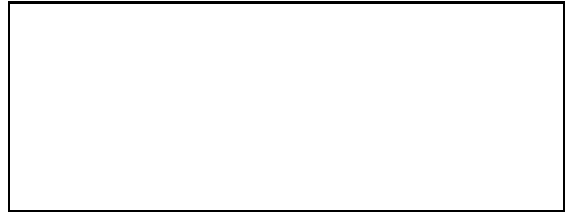
ANSWER:

Problem 8 (8 points) Sketch the polar curve and set up the integral that represents the length of the curve

$$r = \cos(4\theta).$$

(DO NOT EVALUATE THE INTEGRAL)

ANSWER:

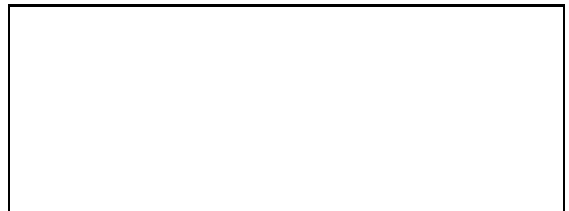


Problem 9 (7 points) Test the following series for convergence or divergence:

$$\sum_{n=1}^{\infty} \frac{n^3}{e^{n^5}}.$$

(DON'T FORGET TO SAY WHICH TEST ARE YOU USING)

ANSWER:



Problem 10 (7 points) Find the radius of convergence and the interval of convergence of the series below. If the interval is finite, do not forget to test the end points.

$$\sum_{n=1}^{\infty} \frac{n}{3^n} (2x - 12)^n.$$

ANSWER:

Problem 11 (8 points)

- (a) Use geometric power series to find the Maclaurin series for $f(x) = \frac{1}{1+x^2}$?
- (b) Use the result from part (a) to find the Maclaurin series for $g(x) = \arctan(x)$, and then find its radius of convergence.

ANSWER:

BONUS problem 1 (2 points) Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{2n} n!}.$$

ANSWER:

Bonus Problem 2 (7 points) Set up, but do not compute, an integral that evaluates the area of the surface of revolution obtained by rotating the infinite curve $y = xe^{-x}$, $1 \leq x$ about the x axis.

ANSWER: