

FINAL EXAM

SPRING 2011

MATH 165

1. (4 points) Show that $\lim_{x \rightarrow 3} (2x + 1) = 7$, using the ϵ - δ definition of limit.

2. (12 points) If the limit exists, find its value. If it does not exist explain WHY.

(a) $\lim_{x \rightarrow -1} \frac{x^2 - 1}{x^2 - x - 2}$

(b) $\lim_{x \rightarrow 1} \frac{\sqrt{5x^2 + 4}}{x^2 + 2}$

(c) $\lim_{x \rightarrow 2^-} \frac{\sqrt{x - 2}}{x^2 + 1}$

(d) $\lim_{x \rightarrow \infty} \frac{-1 + 2x + x^3}{4 - x^2 - 3x^3}$

3. (12 points) Compute the derivatives of the following functions:

(a) $f(x) = \frac{2}{\sqrt{x}} + \tan(5x)$.

(b) $g(x) = \cos(\sin x^2)$.

(c) $h(x) = \int_1^{x^2} \tan t \, dt$.

4. (6 points) Do implicit differentiation to find the slope of the tangent line to the curve

$$y^2 - x + 1 = 0$$

at the point $(2, -1)$.

5. (12 points) Given $f(x) = x^3 - 3x + 2$,

- (a) Where is f increasing?
- (b) Where is f concave up?
- (c) What are the local maxima and minima?

6. (6 points) Find the horizontal and vertical asymptotes of the curve

$$y = \frac{3x^2 + 2x + 1}{x - 1}.$$

Justify your answers with the appropriate limits.

7. (6 points) Find the point on the curve $x^2 - y^2 = 1$ that is closest to $(2, 1)$.

8. (4 points) Set up (but do not compute) the Riemann Sum for the area under the graph of the function $f(x) = x^3 - x$, with $2 \leq x \leq 3$.

Use RIGHT-HAND endpoints and 5 subintervals.

9. (16 points) Evaluate the following integrals:

(a) $\int \cos x \sin^5 x \, dx.$

(b) $\int x \sec(x^2) \tan(x^2) \, dx.$

(c) $\int_0^1 \frac{x}{\sqrt{3x^2 + 1}} \, dx.$

(d) $\int_{-1}^1 \frac{x}{\sqrt{x^4 + 5}} \, dx.$

10. (6 points) Sketch the region enclosed by the curves

$$x = y \quad , \quad x = y^3.$$

Find the area of the region.

11. (8 points) Let A be the region bounded by the graphs of

$$y = \cos x \quad , \quad y = \sin x \quad , \quad \text{the } y\text{-axis.}$$

Set up (but do not compute) the integrals to find the volume of the solid generated by revolving A about

- (a) the x -axis.
- (b) the line $x = -1$.

12. (6 points) A force of 35N is required to hold a spring stretched 10 cm from its natural length. How much work is done when stretching the spring 5 cm further?

Bonus (4 points): Evaluate

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}.$$

Bonus (4 points): Evaluate

$$\int \frac{dx}{(2 + \sqrt{x})^3}.$$

