Convexity and curvature in space-time geometry

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ABSTRACT

Lorentzian geometry is the mathematical foundation of Einstein’s theory of general relativity. Until fairly recently, all results in Lorentzian geometry making use of curvature bounds considered only timelike curvatures and timelike geodesics. A space-time is said to satisfy $R \geq K$ if the sectional curvatures of spacelike planes are bounded below by $K$ and the sectional curvatures of timelike planes are bounded above by $K$. Similarly, one can define $R \leq K$ by reversing the inequalities. These conditions naturally generalize the notion of curvature bounds for Riemannian manifolds to the Lorentzian setting. In this talk, we will introduce some basic ideas from Lorentzian geometry and describe the geometric significance of the conditions $R \geq K$ and $R \leq K$. Finally, we will discuss two new results about convex functions arising from the curvature bound conditions $R \geq 0$ and $R \leq 0$. This is joint work with Stephanie Alexander.