There are 4 problems in the quiz.

(6 points) 1. a) Find the general integral of the equation \( x(2u - x)u_x + y(x + u)u_y = u(2x - u) \) in the domain \( \{(x, y) : x > 0, y > 0\} \).

b) Find the solution of this equation such that \( u(x,1) = 2x \) for \( x > 0 \).

(6 points) 2. a) Find the general integral of the equation \( xu_x + yu_y = xyu \) in the domain \( \{(x, y) : y > 0\} \).

b) Find the solution of this equation for such that \( u(x,1) = 1 \) for \( x > 0 \).

(6 points) 3. Consider the differential equation \( xe^yu_x + ye^u_y = xe^x \). For each of the following cases, determine without solving the differential equation if the initial value problem in the domain \( \{(x, y) : x > 0\} \) has a unique solution, no solutions, or infinitely many solutions, and explain your answers:

a) \( u(x, x) = x \),  b) \( u(y^2, y) = y \),  c) \( u(x, x) = x^2 \).

(6 points) 4. For the initial value problem \( x^2(x + u)u_x - y(x^2 + u^2)u_y = u^2(x + u) \), and \( u(x, 4/x) = f(x) \) for \( x > 0 \), find all functions \( f(x) \) such that the initial value problem has infinitely many solutions.