



Stochastic reconstruction from time series: predictive framework for critical transition in natural systems

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The only way to predict critical transitions, i.e. abrupt overall changes in the system's behavior, is construction and using of the system's model. Prognosis is traditionally based on analysis of first-principle models (FPM) that include a set of equations, which are purported to correctly describe the observed phenomenon. An alternative approach developed in recent years is based on the construction of a universal parameterized (global) model of unknown evolution operator directly from chaotic, nonstationary time series (TS) generated by the system. The last approach is often called global reconstruction of the system's phase space. Both of these approaches are however inadequate from prognostic point of view if the system under investigation is complex, i.e. high-dimensional; clearly the complexity is very prevalent feature of natural systems. In case of the FPM approach persistent obstacle to developing methods for robust prediction of natural system behavior arises from the too high inherent sensitivity of the model to initial data and model parameters. For the second, global reconstruction approach, the main obstacle is due to too large difference between phase space dimension dS of the system that has generated the initial TS, and the embedding dimension dE , i.e. phase space dimension of the global model that reconstructs correctly phase trajectory corresponding to initial TS. According to the Taken's theorem, dE can be more than twice as more as dS (so called overembedding problem). As a result the structure of global model parameter space appears to be complicated insomuch that the model becomes too sensitive to deviation of its parameter values.

It is clear however that in general so robust process as critical transition is resulted from (slow) trend/s of the only/a very few system parameter/s and manifests itself in change of structure of phase sub-spaces that are significantly less dimensional than both system and embedding phase spaces. It means that properly constructed low-dimensional model may predict correctly critical transition in the high-dimensional system. In this work we put forward approach to construction of the proper low-dimensional model from TS generated by the system under investigation. In this stochastic approach, the robust dynamic properties of the high-dimensional system's evolution are described by a few deterministic equations, while other features are considered as a stochastic process. We develop a Bayesian approach for reconstruction of high-dimensional systems from observed time series and, in particular, propose an efficient form of a low-dimensional stochastic model of the evolution operator, and implement it using artificial neural networks. The predictive capabilities of the approach are illustrated on model examples. Thus, we demonstrate successful prognosis of critical transition based on a scalar time series extracted from an attractor with correlation dimension about 3.5, by using a single-variable stochastic model.