

## References for Roland Roeder’s 2023 MINT Mini-Course: “Dynamics of holomorphic and birational group actions on surfaces”.

Some nice general references on the automorphism group dynamics on K3 surfaces are the surveys [3, 6, 7]

### Lecture 1:

- (a) The affine surface  $S_D$  and its automorphisms: [5, 2, 15].
- (b) The smooth compact surfaces that were called  $W_A$  in my course appears on p. 1 of [12] and p. 3 of [3]. A more general discussion of the Wheeler surfaces appears in Section 3.1 of [4].
- (c) The dynamics on the space of pentagons is discussed in the introduction and Section 3.2 of [4].
- (d) The connection between the dynamics on character varieties and the automorphisms of  $S_D$  is described in [5, 2, 15]. See also the paper by Goldman [10] for more information on the Fricke-Klien trace relations and how they are used in character varieties.
- (e) The connection between the Painlevé 6 Monodromy and the dynamics on the affine surface  $S_D$  is discussed in [5, 2, 15].

### Lecture 2:

- (a) Proof that the canonical invariant volume form  $W_A$  is ergodic is a special case of Corollary 3.2 from [1] and Section 6.1 of [6] is a helpful alternative explanation.
- (b) The discussion about the dynamics of the automorphism group of  $S_D$  is from [15].

### Lecture 3:

- (a) This lecture was based entirely on further results on the dynamics of the automorphism group of  $S_D$  is from [15].
- (b) The Zassenhaus Lemma for Lie Groups can be found in Section 5.4 of [13]. A more classical reference is [14].
- (c) The “non-linear Zassenhaus Lemma” dates back to Ghys [9]. It plays an important role in [11] and the version presented here is Proposition 7.1 in [15]. It was recently used to study “canonical currents” in [8].

## REFERENCES

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See also [https://math.uchicago.edu/~sfilip/public\\_files/lectures\\_k3\\_dynamics.pdf](https://math.uchicago.edu/~sfilip/public_files/lectures_k3_dynamics.pdf)
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