Complex Dynamics School: Building a bridge to higher dimensions

IUPUI, March 14-18, 2022.

All talks are in room LD136

Monday 3/14:

9:00am-9:45am: Measure-theoretic entropy;
Vanessa Matus de la Parra (University of Rochester).

10:00am-10:45am: Topological entropy and variational principle;
Malavika Mukundan (University of Michigan).

11:00am-11:45am: Rokhlin-Parry Formula;
Jorge Olivares (University of Rochester).

12:00pm-2:00pm: Lunch Break (see our list of favorite restaurants).

2:00pm-2:45pm: Potential theory and the computation of $dd^c \log|\cdot|$;
Nikolai Prochorov (MSRI).

3:00pm-3:45pm: Lyapunov exponents and Przytycki's formula;
Ricky Simanjuntak (Indiana University, Bloomington).

4:00pm-4:45pm: Fatou Coordinates in one dimension;
Somi Kang (Indiana University, Bloomington).

Tuesday 3/15:

9:00am-9:45am: Equidistribution in dimension 1;
Alex Kapiamba (University of Michigan).

10:00am-10:45am: Complex projective plane and its endomorphisms;
Achinta Kumar Nandi (Oklahoma State University).

11:00am-11:45am: Bezout's theorem;
Max Weinreich (Brown University).

12:00pm-2:00pm: Lunch Break (see our list of favorite restaurants).

2:00pm-2:45pm: Blowups, rational surface automorphisms, and the Cremona involution;
Shengyuan Zhao (SUNY Stony Brook).

3:00pm-3:45pm: Kähler manifolds;
Richard Birkett (Notre Dame).

4:00pm-4:45pm: Currents and pluripotential theory;
Seung uk Jang (University of Chicago).

Wednesday 3/16:

9:00-10:00: Dynamics of complex Hénon mappings, Part 1;
Eric Bedford (SUNY Stony Brook).

10:15-11:15: Dynamics of complex Hénon mappings, Part 2;
Eric Bedford (SUNY Stony Brook).

11:30-12:30: Local Dynamics around a fixed point in several complex variables, Part 1;
Liz Vivas (Ohio State University).

12:30-2:30: Lunch

2:30-3:30: Local Dynamics around a fixed point in several complex variables, Part 2;
Liz Vivas (Ohio State University).

4:00-5:00: Topological entropy and degree of smooth mappings;
Michał Misiurewicz (IUPUI).

Thursday 3/17:

9:00-10:00: Gromov's entropy bound for holomorphic maps, Part 1;
Jeffrey Diller (Notre Dame University).

10:15-11:15: Gromov's entropy bound for holomorphic maps, Part 2;
Jeffrey Diller (Notre Dame University).

11:30-12:30: Measures of maximal entropy for holomorphic endomorphisms of the complex projective plane, Part 1;
Mattias Jonsson (University of Michigan).

12:30-2:30: Lunch

2:30-3:30: Measures of maximal entropy for holomorphic endomorphisms of the complex projective plane, Part 2;
Mattias Jonsson (University of Michigan).

4:00-5:00: Rational surface automorphisms: Real vs. Complex dynamics, Part 1;
Kyounghee Kim (Florida State University).

Friday 3/18:

9:00-10:00: Rational surface automorphisms: Real vs. Complex dynamics, Part 2;
Kyounghee Kim (Florida State University).

10:15-11:15: The structure at infinity for Hénon mappings;
John Hubbard (Cornell University).

11:30-12:30: The Julia sets for Hénon mappings;
John Hubbard (Cornell University).

12:30-infinity: *Lunch and relax.*

Eric Bedford:

Title: Dynamics of complex Hénon mappings.

Abstract: I will talk about the dynamics of complex Hénon mappings.

Jeffrey Diller:

Title: Gromov's entropy bound for holomorphic maps.

Abstract: In these two talks I'll explain an upper bound due to Gromov for the entropy of a holomorphic self-map of a compact Kahler manifold. The arguments are an excellent illustration of why Kahler manifolds and holomorphic maps are special. My plan is to start with the particular case of maps of projective space and expand from there. Time permitting, I'll also discuss Dinh and Sibony's generalization of Gromov's bound from holomorphic maps to rational maps on complex projective varieties.

Mattias Jonsson:

Title: Measures of maximal entropy for holomorphic endomorphisms of the complex projective plane.

Abstract: Holomorphic endomorphisms of the complex projective plane are some of the best behaved discrete-time holomorphic higher-dimensional dynamical systems. To each such endomorphism, Hubbard-Papadopol and Fornaess-Sibony constructed a natural invariant probability measure. I will explain the work of Briend-Duval, who among other things proved that this measure is the unique measure of maximal entropy.

John Hubbard:

Talk 1 title: The structure at infinity for Hénon mappings.

Abstract: A Hénon map is an automorphism of $\mathbb{C}^2$, but as a map $\mathbb{P}^2 \to \mathbb{P}^2$ it is undefined at the point $[0:1:0]$, and collapses the remainder of the line $z = 0$ at infinity to the point $[1:0:0]$. The projective plane $\mathbb{P}^2$ is not the correct compactification of $\mathbb{C}^2$ for Hénon maps: as we will see, there is a lot hiding in that "point of indeterminacy".

Constructing the correct compactification will take us through blow-ups, sequence spaces, real oriented blowups, cohomology of projective limits, etc, and will end up with a 3-sphere containing two linked solenoids. These solenoids are where the angles of external rays live, and give some hope of understanding Hénon mappings the way we understand Julia sets of polynomials.

Talk 2 title: The Julia sets for Hénon mappings.

Abstract: Since Hénon maps are invertible, they have two filled-in Julia sets: the set K^+ of points with bounded forward orbits and the set K^- of points with bounded backwards orbits. Set $K = K^+ \cap K^-$.

For a polynomial, the Juia set is the boundary of the filled-in Julia set. By analogy, define

$$J^+ = \partial K^+, \qquad J^- = \partial K^-, \qquad J = J^+ \cap J^-.$$

These are the sets we really want to understand. Unfortunately, there are very few cases where they are understood: the small perturbations of hyperbolic polynomials. I will give this description, which is already pretty elaborate.

Further, I will describe what may be an approach to describe these sets more generally, in terms of a generalization of the "pinched disk model" which has been so successful for polynomials in one variable, but in several variables appears to be beyond my topological abilities.

Kyounghee Kim:

Title: Rational surface automorphisms: Real vs. Complex dynamics

Abstract: When a rational surface automorphism is given by rational functions of real coefficients, one can consider two related dynamical systems: Real and Complex dynamics. It is interesting to know how those two dynamics are related. We will discuss how to construct such automorphisms using an invariant cubic. To understand the relationship between real dynamics and complex dynamics, we will estimate the topological entropy using the growth rate in the Homology group and the growth rate in the Fundamental group.

Michał Misiurewicz:

Title: Topological entropy and degree of smooth mappings.

Abstract: This is a 45 years old proof that for every smooth mapping of a compact differentiable manifold onto itself, its topological entropy is at least the logarithm of its degree.

Liz Vivas:

Title: Local Dynamics around a fixed point in several complex variables.

Abstract: Given a holomorphic map with a fixed point, we are interested in understanding the dynamics close to that fixed point. We will review the results in one dimension and direct our attention to the several dimensions case. Applications to global dynamics will be given.