The following results are true and you can use them in your solutions of the homework problems, if you wish, even though we will not have time to go over their proofs in class.

Theorem 1. (Fredholm's Alternative (modern version))
If T is in $\mathcal{B}(\mathcal{H})$, then $(im(T))^\perp = ker(T^*)$ and $(im(T^*))^\perp = ker(T)$.

Theorem 2. (Fredholm's Alternative (Classical version)) *Suppose T is in $\mathcal{B}(\mathcal{H})$ and $im(T)$ is closed. For b in $\mathcal{H}$, exactly one of the following is solvable:*

- $Tx = b$.
- $T^*y = 0$ *and* $<y, b> \neq 0$.

There are suitable versions of these results for T in $\mathcal{B}(X,Y)$ for X and Y Banach spaces where *annihilators* (up and down) replace orthogonal complements. These allow the following Banach space (including Hilbert space) results to be obtained as corollaries.

Corollary 3. *If T is in $\mathcal{B}(X,Y)$ for X and Y Banach spaces, then*

- *$ker(T')$ is weak-star closed in Y'.*
- *$im(T)$ is dense in Y if and only if T' is one-to-one.*
- *$im(T')$ is weak-star dense in X' if and only if T is one-to-one.*

The following result concerns conditions for the range of an operator to be closed.

Theorem 4. *Suppose T is in $\mathcal{B}(X,Y)$ for X and Y Banach spaces.*
Then the following conditions are equivalent:

- *$im(T)$ is norm closed in Y.*
- *$im(T')$ is weak-star closed in X'.*
- *$im(T')$ is norm closed in X'.*