NAME: ____________________

Math 35300 (Cowen) **Practice Lab Test** **Spring 2015**

There are 7 questions, 7 pages, and 100 points on this test.

↑↑ In Actual Test, 1 or 2 problems per page!

- For decimal approximations, it is enough to give 2 decimal places in your answers.
- Show your work when working by hand and when using a machine, explain what it did.
- Explain your answers for each question in such a way that your reasoning can be followed!!

(15 points) 1. The following system has infinitely many solutions:

$$\begin{cases} \quad\quad\;\; w + 2x + 3y + 4z = -4 \\ 5v + 6w + 7x + 8y + 9z = 1 \\ v + w + x + y + z = 1 \\ v \quad\quad\; + x \quad\;\; + z = 1 \end{cases}$$

Find the general solution and *find three solutions* explicitly.

(15 points) 2. Consider the following two systems of equations.

$$(H)\begin{cases} a - 2b + c \quad\quad + 3e + f = 0 \\ -a + 3b - 2c - 3d - 2e - 2f = 0 \\ 2a - 5b + 4c + 5d + 4e + 5f = 0 \\ -a + 4b - 2c - 4d - 2e - 2f = 0 \\ a - 3b + 4c + 7d \quad\quad + 5f = 0 \end{cases}$$

and

$$(N)\begin{cases} a - 2b + c \quad\quad + 3e + f = 3 \\ -a + 3b - 2c - 3d - 2e - 2f = -2 \\ 2a - 5b + 4c + 5d + 4e + 5f = 6 \\ -a + 4b - 2c - 4d - 2e - 2f = -1 \\ a - 3b + 4c + 7d \quad\quad + 5f = 3 \end{cases}$$

The vectors $(0, -1, 1, -1, -1, 0)$ and $(0, 1, -1, 1, 1, 0)$ are solutions of system (H).
The vectors $(1, 1, 0, 0, 1, 1)$ and $(-3, 0, 2, -1, 1, 1)$ are solutions of system (N).
Use theorems you know (i.e. do *not* try to solve the systems completely!) to answer the following questions.

(a) Find two linearly independent solutions of (H) different from those given above.
(b) Find two solutions of (N) different from those given above.

(15 points) 3. Let $w_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$, $w_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$, $w_3 = \begin{pmatrix} 1 \\ -5 \\ 3 \\ -1 \end{pmatrix}$, $w_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}$, and $w_5 = \begin{pmatrix} -2 \\ 1 \\ 1 \\ -1 \end{pmatrix}$.

(a) Explain how you can tell that $\{w_1, w_2, w_3, w_4, w_5\}$ is a linearly dependent set without doing any calculations.
(b) Write one of the vectors as a linear combination of the rest.

(15 points) 4. Let $p_1 = (1, -1, 1, 1, 1)$, let $p_2 = (1, 1, 0, -2, 2)$, and let $q = (5, 1, 2, -4, 8)$.

(a) Write q as a linear combination of p_1 and p_2.

(b) Let $r = (8, -2, -4, 5, 1)$. Write r as a linear combination of p_1, p_2, and q or show that it is not a linear combination of them.

(c) What is the dimension of the subspace spanned by p_1, p_2, q, and r? ____________
Explain your answer!

(10 points) 5. Let $\mathcal{W}$ be the column space of F, the subspace spanned by the columns of F, where

$$F = \begin{pmatrix} 2 & 1 & 1 & 1 \\ -1 & -1 & -2 & 1 \\ 3 & 1 & 0 & -2 \\ 1 & 1 & 2 & 1 \end{pmatrix}$$

(a) Choose some of the column vectors of F to get a basis for $\mathcal{W}$ and explain why your answer is correct.

(b) If $\mathcal{W} = \mathbb{R}^4$, explain why this is true.
If $\mathcal{W}$ is *NOT* $\mathbb{R}^4$, find a vector x in $\mathbb{R}^4$ that is *NOT* in $\mathcal{W}$ and explain why it is not.

(15 points) 6. Suppose the vectors $\{u_1, u_2, u_3, u_4\}$ are a basis for $\mathbb{R}^4$.
Show that if $y_1 = 2u_1 - 3u_4$, $y_2 = u_1 + 2u_2 - u_3$, $y_3 = u_2 + 2u_3 - u_4$ and $y_4 = u_1 + 2u_2 - u_3 + u_4$, then $\{y_1, y_2, y_3, y_4\}$ is also a basis for $\mathbb{R}^4$.

(15 points) 7. (a) Prove the following theorem:

Let Q be an $m \times n$ matrix. If P is an $n \times m$ matrix so that $PQ = I$, then $\mathcal{N}(Q) = (0)$ (that is, the null space of Q is the zero subspace).

(b) Prove the following theorem:
(You may use the result of part (a) even if you did not answer part (a)!)

Let Q be an $m \times n$ matrix.
If P is an $n \times m$ matrix so that $PQ = I$, then the columns of Q are linearly independent.