Math 300 **Due October 30**

Homework 8

Definition A finite set of integers is said to be *a set of consecutive integers* if there are integers m and k so that the given integers are m, $m+1$, $m+2$, $\cdots$, $m+k$.

For example, the set $\{17, 18, 19, 20\}$ is said to be a set of four consecutive integers because the set consists of 4 integers and (using $m = 17$ and $k = 3$ in the definition) the integers are 17, $17+1$, $17+2$, and $17+3$.

Reading: Theorem: If n is the sum of five consecutive integers, then n is divisible by 5. Conversely, if n is divisible by 5, then n is the sum of five consecutive integers.

Proof: Suppose n is the sum of five consecutive integers. Then there is an integer m so that n is the sum of the integers m, $m+1$, $m+2$, $m+3$ and $m+4$, that is, we have

$$n = m + (m+1) + (m+2) + (m+3) + (m+4)$$

Removing the parentheses and rearranging the terms, this means,

$$n = m + m + m + m + m + 1 + 2 + 3 + 4 = 5m + 10$$

In particular, this gives $n = 5m + 10 = 5(m+2)$ and, since $m+2$ is an integer, this shows n is divisible by 5.

Conversely, suppose n is divisible by 5, say $n = 5a$. Let $m = a - 2$ so $a = m + 2$. Then

$$n = 5(m+2) = 5m+10 = m+m+m+m+m+1+2+3+4 = m+(m+1)+m+2)+(m+3)+(m+4)$$

which shows that n is sum of five consecutive integers.

1. Write 1245 as the sum of five consecutive integers.

2. (a) Prove: If n is the sum of 4 consecutive integers, then n is not divisible by 4.

(b) Write 1346 as the sum of four consecutive integers.

3. Suppose x and y are odd integers. Show that $x^2 + y^2$ is not a perfect square, that is, that $x^2 + y^2$ is not the square of an integer.

4. Find a polynomial, p, of degree three or less, that satisfies $p(-1) = 4$, $p(2) = 3$, $p(4) = 0$, and $p(5) = -1$.

5. Prove: If a, b, and c are distinct real numbers and f and g are polynomials of degree 2 that satisfy $f(a) = g(a)$, $f(b) = g(b)$, and $f(c) = g(c)$, then f and g are actually the same polynomial.

6. Although, generally, it requires a polynomial of degree 3 to interpolate four points, there is a polynomial, q, of degree 2 that satisfies $q(-1) = 9$, $q(1) = 3$, $q(2) = 6$, and $q(3) = 13$. Find a polynomial, q, of degree 2 that solves this interpolation problem.