Math 300 **Due October 23**

Homework 6

Read: DoA, Chapter 2, section 4 (that is, the first section of Chapter 2)

1. Let $f(x) = 2x^3 - 3x^2 + 4x - 5$, let $g(x) = 4x^2 - 5x - 2$, and let $h(x) = x^3 - 7x$.

(a) For the polynomials f, g, and h: What are their degrees? What are their leading coefficients? What are their coefficients 'a_2'? What are their constant coefficients or their free terms?

(b) Find $f + g$, $g - h$, $f - 2h$, and gh.

(c) What are the degrees of $f + g$, $g - h$, $f - 2h$, and gh?

2. Use Theorem 11 of Chapter 1 of *Discourses on Algebra* to find all the rational roots of these polynomials:

$(a)\quad x^2 + 24x + 63$ $\qquad (b)\quad x^3 - 37x + 84$

$(c)\quad x^3 - 42x + 49$ $\qquad (d)\quad x^4 + 118x - 35$

3. (a) For which degrees and which integer values of a does the polynomial $x^n + ax + 1$ have rational roots?

(b) For which integer values of a does the polynomial $x^4 + ax + 2$ have rational roots?

(c) For which integer values of a does the polynomial $x^4 + ax + 6$ have rational roots?

4. Prove the following extension of Theorem 11 of Chapter 1 of *Discourses on Algebra*: Suppose f is a polynomial with integer coefficients. If α is a rational root of f, and α is written as $\alpha = \frac{p}{q}$ where p and q are integers, with $q > 0$ and p and q have no common factors, then q divides the leading coefficient of f and p divides the constant coefficient of f.

5. Use the exercise above to find all rational roots of $2x^3 + x^2 - 16x - 15$