**Math 300** **Due October 18**

## Homework 5

**1.** (*Signature for divisibility by* 11) Suppose an integer $n$ has decimal representation $n = d_{2j}d_{2j-1}d_{2j-2}\cdots d_4d_3d_2d_1d_0$. The *signature* (or *signature for divisibility by* 11) of $n$ is

$$s = d_{2j} - d_{2j-1} + d_{2j-2} - \cdots + d_4 - d_3 + d_2 - d_1 + d_0$$

Prove that $n$ is divisible by 11 if and only if $s$, the signature of $n$, is divisible by 11. (For example, we can tell that $12,356,322$ is divisible by 11 because

$$-1 + 2 - 3 + 5 - 6 + 3 - 2 + 2 = 12 - 12 = 0$$

is divisible by 11.)

**2.** (More on *signature divisibility by* 11) Use the ideas of Exercise 1 and "casting out nines" to show that the remainder when dividing $n$ by 11 is the same as the remainder when dividing $s$, the signature of $n$, by 11. (The notation in this exercise is the same as in Exercise 1.)

**3.** Use the Theorem from the reading on the test and the observation

$$2(12641) = 25282 = (159)^2 + 1$$

to write 12641 as the sum of the squares of two integers.

**4.** For which integers, $n$, is $n^2 + 3n + 8$ divisible by 2?

**5.** For which integers, $n$, is $n^2 + 3n + 8$ divisible by 3?

**6.** For which integers, $n$, is $n^2 + 3n + 8$ divisible by 4?