Math 300 **Due November 27**

Homework 13

1. The Fibonacci Sequence is 1, 1, 2, 3, 5, 8, 13, 21, $\cdots$. It is given recursively by $a_1 = 1$, $a_2 = 1$, and $a_{n+1} = a_n + a_{n-1}$.

(a) Show from the recursion relation above that the ratio $r_n = \frac{a_{n+1}}{a_n}$ satisfies the recursion relations: $r_1 = 1$ and $r_{n+1} = 1 + \frac{1}{r_n}$

(b) Prove (by induction) that $1 \leq r_n \leq 2$ for all positive integers n.

(c) Prove (by induction) that $|r_{n+2} - r_{n+1}| \leq |r_{n+1} - r_n|$.

Not for grading: If you take real analysis, you will learn that (a), (b), and (c) imply that the sequence $\{r_n\}_{n=1}^{\infty}$ converges. If you have had Math 164, you probably learned how to find the limit of the sequence of ratios; can you find the limit?

2. A number n is called a *triangular number* if a triangle of touching pennies can be made that has n pennies in it. The first four triangular numbers are 1, 3, 6, and 10, being made from triangles of 1, 2, 3, and 4 on a side. Find an expression for the n^{th} triangular number and prove (by induction) that your formula is correct.

3. A number n is called a *hexagonal number* if a hexagon of touching pennies can be made that has n pennies in it. The first two hexagonal numbers are 1 and 7 being made from hexagons of 1 and 2 on a side. Find an expression for the n^{th} hexagonal number and prove (by induction) that your formula is correct.

4. A number n is called a *cannonball number* if a stack of touching cannonballs (with a triangular base) can be made that has n cannonballs in it. The first three cannonball numbers are 1, 4, and 10 being stacks of cannonballs with 1, 2, and 3 layers. Find an expression for the n^{th} cannonball number and prove (by induction) that your formula is correct.

5. A rectangular swimming pool is 15 feet by 20 feet. A walk around the edge of the pool is made by putting tiles that are one foot square around the outside of the pool. For example, if one row of tiles is put around the outside, we will have walk one foot wide around the outside made from 74 tiles. Find an expression for the number of tiles needed to make a walk around the outside that is n feet wide and prove (by induction) that your formula is correct.