Math 300 **Due November 6**

Homework 12

1. Let $f(x)$ be a polynomial with *rational* coefficients. Prove that there exists a polynomial $g(x)$ (which depends on $f(x)$!) with *integer* coefficients such that $f(\alpha) = 0 \Rightarrow g(\alpha) = 0$.

2. If α is a possible value for x, and $f(x) = a_0 + a_1x + \ldots + a_nx^n$, prove, using polynomial division (that is, one step of the Euclidean Algorithm), that the remainder when dividing $f(x)$ by $(x - \alpha)$ is $f(\alpha)$.

3. Find the greatest common divisor of the following pairs of polynomials:

(a) $f(x) = 6x^4 - 18x^2 + 12$
$g(x) = 15x^4 - 15x^3 - 60x^2 + 30x + 60$

(b) $f(x) = x^4 - 5x^3 + 8x^2 - 10x + 12$
$g(x) = 2x^4 - 12x^3 + 20x^2 - 24x + 32$

(c) $f(x) = x^3 - 3x^2 + 3x - 9$
$g(x) = x^5 + x^4 - x^3 - x^2 - 12x - 12$

4. Determine if $(x - \alpha)$ is a factor of $f(x)$ for the following values of α and polynomials $f(x)$:

(a) $\alpha = \frac{1}{2}, f(x) = \frac{16}{175}(x^3 - 2x^2 - 5x + 6)$

(b) $\alpha = -2, f(x) = 2x^3 + 6x^2 - 2x - 6$

(c) $\alpha = 0, f(x) = g(x) + 1$ where $g(0) \neq -1$

(d) $\alpha = i, f(x) = x^4 - x^2 - 2$ where $i = \sqrt{-1}$