Math S166 **Due February 6**

Homework S3

In the handout on inverse functions, we showed that if a function is continuous and one-to-one on an interval, then it is strictly increasing or strictly decreasing. The following is a converse, but does not require the continuity.

1. Suppose h is defined on the interval I and strictly increasing on that interval. Prove that h is one-to-one on I.

In the following, we will invent a new function and develop some of its properties.

Define the function S by, for x a real number,

$$S(x) = \int_0^x \frac{dt}{\sqrt{t^2+1}}$$

Using the Riemann sums for this integral (with $n = 100$), it follows that $S(1) = .881$ to three decimal places.

2. Explain why S is defined for every real number. This shows that the domain of S is $\mathbb{R}$.

3. Find $S'(x)$ and $S''(x)$ and use your results to show that S is strictly increasing on $\mathbb{R}$.

4. Find a relationship between $S(x)$ and $S(-x)$.

5. Show that $\sqrt{t^2+1} < t+1$ for $t > 0$ and use the inequality to show that $S(x) > \ln(x+1)$ for $x > 0$.

6. Find $\lim_{x\to\infty} S(x)$ and $\lim_{x\to-\infty} S(x)$. What is the range of S, that is, what is the set $\{y : y = S(x) \text{ for some } x \text{ in } \mathbb{R}\}$?

7. Use the results of the previous exercises to draw a graph of S.

8. Use an argument similar to that of Exercise 5 above to see that there is a constant C so that $S(x) < C + \ln(x-1)$ for $x \geq 2$.

Challenge Problem:

(This problem will never be assigned or collected. There are solutions that are easy to understand, but there are no solutions that are easy to find!)

Find a function f that maps $[0, 1]$ one-to-one and onto $(0, 1)$.