NAME: _____

Math 165 (Cowen) **Test 5** **27 April 2009**

There are 6 pages, 9 questions, and 100 points on this test. **The test finishes at 1:15pm!** Follow the instructions for each question and show enough of your work that I can understand what you are doing.

(10 points) **1.** Find the area in first quadrant (as a number) bounded by the curve $y = 9x - x^3$.

(10 points) **2.**(a) In terms of areas, what does the integral $\int_0^{2\pi} \sin(x)\,dx$ represent?

(b) Find the area, as a positive number, *between* the curve $y = \sin(x)$ and the x-axis (that is, $y = 0$), between $x = 0$ and $x = 2\pi$. (For example, this would represent the area of carpet needed to cover a part of the floor in the shape of the region between the x-axis and the curve.)

(20 points) **3.**(a) Use integration based on vertical rectangles to find the area, as a number, between the curves $x^2 + y = 3$ and $2x - y = 0$. Show the integral(s) explicitly, as well as the calculations, used to find your answer.

(b) Use integration based on horizontal rectangles to find the area, as a number, between the curves $x^2 + y = 3$ and $2x - y = 0$. Show the integral(s) explicitly, as well as the calculations, used to find your answer.

(10 points) **4.** Find an approximate value for the integral

$$\int_1^3 \frac{3x}{x+2}\,dx$$

by calculating a Riemann sum using 6 subintervals and the right end points of each of the subintervals for evaluating the function.

(10 points) **5.** Charles is thinking about the integral

$$\int_0^2 \frac{r}{4r^4-4r^2+2}\,dr = \int_0^2 \frac{r}{(2r^2-1)^2+1}\,dr$$

and decides it would be easier if he made a substitution. Do the substitution $p = 2r^2 - 1$, that is, find a definite integral with the new variable p that gives the same calculation as the integral above after making the substitution $p = 2r^2 - 1$. (Do NOT evaluate either integral.)

(10 points) **6. Set up** an integral (you do NOT need to evaluate the integral) to find the volume (a paraboloid) obtained by revolving, around the y-axis, the region in the plane bounded by $y = 2x^2$ and $y = 8$ and having $x \geq 0$. (Note that the regions in problems 6 and 7 are the same.)

(10 points) **7. Set up** an integral (you do NOT need to evaluate the integral) to find the volume (a cylinder with hole in the center) obtained by revolving, around the x-axis, the region in the plane bounded by $y = 2x^2$ and $y = 8$ and having $x \geq 0$. (Note that the regions in problems 6 and 7 are the same.)

(10 points) **8.** Find the function f that satisfies $f'(x) = 6x^2 + \sqrt{5x-1}$ and $f(2) = 22$.

(10 points) **9.** $h(x) = \int_{2x}^{x^2+1} \sqrt{t^3+2}\, dt$

Find $h'(x)$.

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(10 points) **1.** Find the area in first quadrant (as a number) bounded by the curve $y = 4x - x^3$.

(10 points) **2.** Find an approximate value for the integral

$$\int_1^3 \frac{3x}{x+2}\,dx$$

by calculating a Riemann sum using 6 subintervals and the left end points of each of the subintervals for evaluating the function.

(20 points) **3.**(a) Use integration based on vertical rectangles to find the area, as a number, between the curves $x^2 + y = 4$ and $3x - y = 0$. Show the integral(s) explicitly, as well as the calculations, used to find your answer.

(b) Use integration based on horizontal rectangles to find the area, as a number, between the curves $x^2 + y = 4$ and $3x - y = 0$. Show the integral(s) explicitly, as well as the calculations, used to find your answer.

(10 points) **4.**(a) In terms of areas, what does the integral $\int_0^{2\pi} \sin(x)\,dx$ represent?

(b) Find the area, as a positive number, *between* the curve $y = \sin(x)$ and the x-axis (that is, $y = 0$), between $x = 0$ and $x = 2\pi$. (For example, this would represent the area of carpet needed to cover a part of the floor in the shape of the region between the x-axis and the curve.)

(10 points) **5.** Charles is thinking about the integral

$$\int_0^3 \frac{p}{4p^4 - 4p^2 + 2}\,dp = \int_0^3 \frac{p}{(2p^2-1)^2 + 1}\,dp$$

and decides it would be easier if he made a substitution. Do the substitution $r = 2p^2 - 1$, that is, find a definite integral with the new variable r that gives the same calculation as the integral above after making the substitution $r = 2p^2 - 1$. (Do NOT evaluate either integral.)

(10 points) **6. Set up** an integral (you do NOT need to evaluate the integral) to find the volume (a paraboloid) obtained by revolving, around the y-axis, the region in the plane bounded by $y = 3x^2$ and $y = 12$ and having $x \geq 0$. (Note that the regions in problems 6 and 7 are the same.)

(10 points) **7. Set up** an integral (you do NOT need to evaluate the integral) to find the volume (a cylinder with hole in the center) obtained by revolving, around the x-axis, the region in the plane bounded by $y = 3x^2$ and $y = 12$ and having $x \geq 0$. (Note that the regions in problems 6 and 7 are the same.)

(10 points) **8.** Find the function f that satisfies $f'(x) = 6x + \sqrt{2x+1}$ and $f(4) = 50$.

(10 points) **9.** $h(x) = \int_{2x}^{x^3+1} \sqrt{t^2+2}\, dt$

Find $h'(x)$.