

The following problems concern the ‘Cantor function’, φ , as described in A-List Problem 4.

1. Find $\varphi(2/3)$, $\varphi(1/3)$, $\varphi(1/4)$, $\varphi(3/4)$, $\varphi(1/2)$, and $\varphi(4/9)$.
2. Recall that if S is a set, $\varphi^{-1}(S) = \{x : \varphi(x) \in S\}$.
Find $\varphi^{-1}(\{1/3\})$, $\varphi^{-1}(\{3/7\})$, $\varphi^{-1}(\{5/7\})$, $\varphi^{-1}(\{1/2\})$, $\varphi^{-1}(\{3/4\})$, and $\varphi^{-1}(\{7/8\})$.
3. Prove: If $0 < x < y < 1$, then $\varphi(x) \leq \varphi(y)$.