Math 35300 (Cowen) **Handout 3** **12 March 2011**

DUE Thursday, 24 March:

(A) Find a formula $T((a,b)) = (?,?)$ for the linear transformation on $\mathbb{R}^2$ that satisfies $T((2,1)) = (2,1)$ and $T((1,-2)) = (0,0)$. Can you identify this transformation geometrically?

(B) Explain why there is no linear transformation on $\mathbb{R}^2$ that satisfies $T((2,1)) = (3,-1)$ $T((1,-2)) = (1,1)$, and $T((1,1)) = (2,0)$. Can you find a (non-linear) continuous function of $\mathbb{R}^2$ into itself that satisfies these conditions?

(C) In the vector space $\mathbb{R}^2$, let R be the linear transformation of reflection in the y-axis and S be the transformation of reflection through the line $x = y$. Find matrices for R and S with respect to the usual basis.

(D) Let $T\colon \mathbb{C}^3 \longrightarrow \mathbb{C}^2$ be defined by $T((x,y,z)) = (x + 2y - z, -2x - 4y + 2z)$.
  (a) Find the matrix for T with respect to the usual bases.
  (b) Find a basis for the kernel of T.
  (c) Find a basis for the range of T.

(E) Let $v_1 = (1,2)$ and let $v_2 = (2,3)$, and let S be the linear transformation on $\mathbb{C}^2$ defined by $S(v_1) = -v_1 + 4v_2$ and $S(v_2) = v_1 - v_2$.
  (a) Find the matrix for S with respect to the basis $\{v_1, v_2\}$.
  (b) Let $w = v_1 + 3v_2$. Find $S(w)$, and express your answer in terms of v_1 and v_2.
  (c) Find the matrix for S with respect to the standard basis.
  (d) Express $S(w)$ in terms of the standard basis.

(F) Let $\mathcal{V}$ be the subspace of $\mathcal{C}$ whose basis is the set of functions 1, $\sin x$, $\cos x$, $\sin 2x$, and $\cos 2x$. (Recall $\mathcal{C}$ is the vector space of continuous functions on $[-\pi, \pi]$.) Find the matrix for the differentiation operator with respect to this basis.

(G) Let B be the linear transformation on $\mathbb{C}^3$ of multiplication by the matrix

$$\begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 3 & -1 & 2 \end{pmatrix}$$

  (a) What is the matrix for the linear transformation B with respect to the usual basis?
  (b) What is the matrix for the linear transformation B with respect to the basis $u_1 = (1,-1,2)$, $u_2 = (-1,2,-1)$, and $u_3 = (0,2,1)$?

(H) Letting u_1, u_2, u_3 be the basis of Exercise (G), let L be the linear transformation on $\mathbb{C}^3$ given by $Lu_1 = 2u_1 + u_2 - u_3$, $Lu_2 = -u_2 + 3u_3$, and $Lu_3 = -4u_3$.
  (a) What is the matrix for the linear transformation L with respect to the basis u_1, u_2, u_3?
  (b) What is the matrix for the linear transformation L with respect to the usual basis?