## Homework 13

1. Let $u=(1,1,1,1,1), v=(0,1,0,1,0), w=(1,0,1,0,1), x=(1,1,0,0,0)$, and $y=(0,0,1,1,1)$. Some of these vectors are eigenvectors for the matrix

$$
E=\left(\begin{array}{rrrrr}
6 & -3 & -1 & 4 & -2 \\
1 & 2 & -1 & 2 & 0 \\
3 & -3 & 2 & 4 & -2 \\
1 & -1 & -1 & 5 & 0 \\
2 & -2 & -1 & 3 & 2
\end{array}\right)
$$

For each vector, decide if it is an eigenvector of $E$ or not, and if it is, find the corresponding eigenvalue.
2. The eigenvalues of

$$
P=\left(\begin{array}{rr}
2 & 4 \\
1 & -1
\end{array}\right)
$$

are 3 and -2 . Find an eigenvector for each eigenvalue of $P$.
3. The eigenvalues of

$$
Q=\left(\begin{array}{rrr}
-4 & -12 & -6 \\
3 & 8 & 3 \\
-3 & -6 & -1
\end{array}\right)
$$

are -1 and 2 . Find a basis for each eigenspace of $Q$.

Find the eigenvalues and bases for the eigenspaces for each of the following matrices.
4. $\left(\begin{array}{rr}2 & 3 \\ -1 & 6\end{array}\right)$
5. $\left(\begin{array}{rr}8 & -10 \\ 5 & -7\end{array}\right)$
6. $\left(\begin{array}{rrr}0 & 2 & -4 \\ 2 & -3 & -2 \\ -4 & -2 & 0\end{array}\right)$
7. $\left(\begin{array}{rrr}-8 & 14 & 22 \\ -4 & 4 & 8 \\ -1 & 4 & 5\end{array}\right)$
8. $\left(\begin{array}{rrr}2 & 1 & 0 \\ -1 & 1 & 1 \\ -3 & -4 & 0\end{array}\right)$
9. $\left(\begin{array}{rrr}2 & -5 & -4 \\ -2 & 3 & 3 \\ 4 & -8 & -7\end{array}\right)$
10. Find the eigenvalues and bases for the eigenspaces of the matrix

$$
\left(\begin{array}{rrrrr}
36 & -24 & 3 & -11 & 4 \\
49 & -34 & 1 & -18 & 6 \\
-25 & 15 & -4 & 5 & -1 \\
13 & -6 & 5 & 1 & -1 \\
37 & -24 & 1 & -12 & 2
\end{array}\right)
$$

Be careful to interpret the answers given by your machine correctly!
11. (a) Prove that if $E$ is an $n \times n$ matrix, then the eigenvalues of $E^{t}$, the transpose of $E$, are the same as those of $E$.
(b) Give an example of a $2 \times 2$ matrix, $E$, such that the eigenvectors of $E^{t}$ are different from those of $E$.

