Homework 13

1. Let u = (1, 1, 1, 1, 1), v = (0, 1, 0, 1, 0), w = (1, 0, 1, 0, 1), x = (1, 1, 0, 0, 0), and y = (0, 0, 1, 1, 1). Some of these vectors are eigenvectors for the matrix

$$E = \begin{pmatrix} 6 & -3 & -1 & 4 & -2 \\ 1 & 2 & -1 & 2 & 0 \\ 3 & -3 & 2 & 4 & -2 \\ 1 & -1 & -1 & 5 & 0 \\ 2 & -2 & -1 & 3 & 2 \end{pmatrix}$$

For each vector, decide if it is an eigenvector of E or not, and if it is, find the corresponding eigenvalue.

2. The eigenvalues of

$$P = \left(\begin{array}{cc} 2 & 4\\ 1 & -1 \end{array}\right)$$

are 3 and -2. Find an eigenvector for each eigenvalue of P.

3. The eigenvalues of

$$Q = \left(\begin{array}{rrr} -4 & -12 & -6 \\ 3 & 8 & 3 \\ -3 & -6 & -1 \end{array}\right)$$

are -1 and 2. Find a basis for each eigenspace of Q

Find the eigenvalues and bases for the eigenspaces for each of the following matrices.

 $4. \left(\begin{array}{cc} 2 & 3 \\ -1 & 6 \end{array}\right)$

 $5. \left(\begin{array}{cc} 8 & -10 \\ 5 & -7 \end{array}\right)$

 $6. \left(\begin{array}{rrr} 0 & 2 & -4 \\ 2 & -3 & -2 \\ -4 & -2 & 0 \end{array}\right)$

 $7. \left(\begin{array}{rrr} -8 & 14 & 22 \\ -4 & 4 & 8 \\ -1 & 4 & 5 \end{array} \right)$

 $8. \left(\begin{array}{rrr} 2 & 1 & 0 \\ -1 & 1 & 1 \\ -3 & -4 & 0 \end{array}\right)$

- $9. \left(\begin{array}{rrr} 2 & -5 & -4 \\ -2 & 3 & 3 \\ 4 & -8 & -7 \end{array}\right)$
- 10. Find the eigenvalues and bases for the eigenspaces of the matrix

$$\begin{pmatrix}
36 & -24 & 3 & -11 & 4 \\
49 & -34 & 1 & -18 & 6 \\
-25 & 15 & -4 & 5 & -1 \\
13 & -6 & 5 & 1 & -1 \\
37 & -24 & 1 & -12 & 2
\end{pmatrix}$$

Be careful to interpret the answers given by your machine correctly!

- 11. (a) Prove that if E is an $n \times n$ matrix, then the eigenvalues of E^t , the transpose of E, are the same as those of E.
 - (b) Give an example of a 2×2 matrix, E, such that the eigenvectors of E^t are different from those of E.