## Homework 10

1. (a) Use the Theorem on orthogonal sets to show that the vectors $w_1 = (1, 1, 0)$, $w_2 = (1, -1, 1)$, and $w_3 = (-1, 1, 2)$ are a basis for $\mathbb{R}^3$.
   (b) Use the corresponding expansion theorem to write $v = (2, -1, 3)$ as a linear combination of $w_1$, $w_2$, and $w_3$.

2. The vectors $u_1$, $u_2$, $u_3$, $u_4$ are orthogonal vectors that span the subspace $\mathcal{U}$ of $\mathbb{R}^{11}$.
   Moreover, $\|u_1\| = 1$, $\|u_2\| = 2$, $\|u_3\| = 3$, and $\|u_4\| = 1$.
   (a) What is the dimension of the subspace $\mathcal{U}$?
   (b) Find $\|v\|$ for $v = 3u_1 - 2u_2 + 4u_3 - u_4$?

3. The vectors $u$, $v$, and $w$ are in $\mathbb{R}^n$ and we are given that $\|u\| = 1$, $\|v\| = 2$, $\|w\| = 3$, that $\langle u, v\rangle = -1$, $\langle u, w\rangle = 2$, and that $w$ is perpendicular to $v$.
   (a) Find $\|u - 3v + 2w\|$.
   (b) Show that $u$, $v$, and $w$ are linearly independent.

4. (a) Show that $v_1 = (1, 1, 1, 1)$; $v_2 = (1, 1, -1, -1)$; $v_3 = (1, -1, 1, -1)$; and $v_4 = (1, -1, -1, 1)$ form an orthogonal basis for $\mathbb{R}^4$.
   (b) Write $w = (2, 1, -1, 2)$ as a linear combination of $v_1$, $v_2$, $v_3$, and $v_4$.

5. (a) Show that $v_1 = (1, -1, 1)$ and $v_2 = (3, 2, -1)$ are orthogonal vectors in $\mathbb{R}^3$.
   (b) Is $w = (2, 1, -1)$ in the subspace spanned by $v_1$ and $v_2$?
   (c) Find a non-zero vector in $\mathbb{R}^3$ that is perpendicular to each of $v_1$ and $v_2$.

6. (a) Show that $v_1 = (1, 0, 1, 1)$; $v_2 = (1, 1, -1, 0)$; and $v_3 = (1, -1, 0, -1)$ are orthogonal vectors in $\mathbb{R}^4$.
   (b) Is $w = (6, -1, 2, 1)$ in the subspace spanned by $v_1$, $v_2$, and $v_3$?
   (c) Find a non-zero vector in $\mathbb{R}^4$ that is perpendicular to each of $v_1$, $v_2$, and $v_3$.

7. The vectors $v_1 = (1, 1, -1)$; $v_2 = (2, 1, 2)$; and $v_3 = (2, -1, -1)$ are a basis for $\mathbb{R}^3$. Use the Gram–Schmidt orthogonalization process to create an orthonormal basis for $\mathbb{R}^3$.

8. The vectors $v_1 = (1, 1, -1, 1)$; $v_2 = (1, 0, 1, 2)$; $v_3 = (1, -2, -1, 0)$; and $v_4 = (0, 2, 1, -1)$ are a basis for $\mathbb{R}^4$. Use the Gram–Schmidt orthogonalization process to create an orthonormal basis for $\mathbb{R}^4$.

9. The vectors $v_1 = (1, 0, -1)$ and $v_2 = (2, 1, -1)$ span the subspace $\mathcal{U}$ in $\mathbb{R}^3$. Use the Gram–Schmidt orthogonalization process to create an orthonormal basis for $\mathcal{U}$.

10. The vectors $v_1 = (1, 0, -1, 1)$; $v_2 = (2, 1, 1, -1)$; and $v_3 = (1, -1, -1, 0)$ span the subspace $\mathcal{W}$ in $\mathbb{R}^4$. Use the Gram–Schmidt orthogonalization process to create an orthonormal basis for $\mathcal{W}$.

11. Let $\mathcal{W}$ be the hyperplane (i.e. 3 dimensional subspace) in $\mathbb{R}^4$ with equation $2a + b - c + 2d = 0$. Find an orthonormal basis for $\mathcal{W}$.