NAME: ____________________________

Math 276 (Cowen) **Homework 3** **Due 27 February**

Follow the instructions for each question and show enough of your work that I can understand what you are doing.

(10 points) **1.** Show by using truth tables that the statements $p \to (q \vee \neg r)$ and $\neg p \vee (q \vee \neg r)$ are equivalent.

(10 points) **2.** Write the negation of the statement

$$(\exists x)\,(p(x) \wedge q(x)) \to r(x)$$

in a way in which '¬' is not a main connective
(that is, '¬' does not apply to a compound statement).

3. For this problem, the universal set is the set of positive integers, $\mathbf{N}$.

Let $B = \{n : 1 \leq n \leq 60\}$.

Let $A_2 = \{2k : k \in \mathbf{N}\}$, let $A_3 = \{3k : k \in \mathbf{N}\}$, and let $A_5 = \{5k : k \in \mathbf{N}\}$. Recalling that $\overline{C}$ is the complement of the set C, find the set

$$B \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_5}$$

4. Prove, using induction, that for all positive integers n, that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$

5. Define a sequence $\{a_n\}$ by $a_1 = 3$ and for each positive integer n, $a_{n+1} = 2a_n + 1$.

(a) Find the first five terms in the sequence: a_1, a_2, a_3, a_4, and a_5.

(b) Prove by induction that $a_n > 2^n$ for every positive integer n.

6.

(a) **For which integers, n, is the integer $n^2 + 4n + 3$ divisible by 2.**

(b) **For which integers, n, is the integer $n^2 + 4n + 3$ divisible by 3.**

7. Let $\mathbf{N}$ be the set of positive integers and let $T = \{m \in \mathbf{N} : m \text{ is not divisible by } 3\}$. We know the set T is countable because it is a subset of the integers.

(a) Find a one-to-one correspondence between $\mathbf{N}$ and T by explicitly describing a function $f : \mathbf{N} \mapsto T$. (Hint: One way to do this is by mapping the odd integers to the numbers that have remainder 1 when divided by 3 and mapping the even integers to numbers that have remainder 2 when divided by 3.)

(b) For the function f you defined in (a) above, what are $f(47)$ and $f(34)$?

(c) For the function f you defined in (a) above, for which n is $f(n) = 121$? for which n is $f(n) = 98$?