

Homework Supplement 1

Definition If a and b are integers, $a \neq 0$, we say b is *divisible by* a or a *divides* b if there is an integer x so that $b = ax$.

1. In the following statements, suppose a , b , c , x , and y are integers.
 - (a) Show that if a divides b , then a divides bc .
 - (b) Prove that if a divides b and b divides c , then a divides c .
 - (c) Show: If a divides b and a divides c , then a divides $bx + cy$ for any integers x and y .

2. Use the fact that every integer is either even or it is odd to show that for all integers, n , the number $n^2 - n$ is divisible by 2.

3. Show that for each integer n , the number $n^3 - n$ is divisible by 3.

4. Give a direct proof of the assertion on the “Number Assumption” handout that if a and b are rational numbers, there is a rational number c so that $a < c < b$.

Note: The words “prove”, “show”, “demonstrate”, etc. all mean the same thing in a math class.