Let's Make a Deal!

The Monty Hall Problem

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Game Theory

The study of interacting decisions of rational agents and how these strategies produce outcomes with respect to the utilities of the agents.

Assume...

- Players are *rational*

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Assume...

- Players are *rational*
- Players always have a set of preferences and always act in order to maximize these preferences

Games

- situations in which players act and respond to each other with utility-maximizing strategies

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Nash Equilibrium

- refers to whole strategies for each player that maximize the utility of every player

First Choice	Prize Door	Switch	Stay
1	1	L	W
1	2	W	L
1	3	W	L
2	1	W	L
2	2	L	W
2	3	W	L
3	1	W	L
3	2	W	L
3	3	L	W

Table: Should You Switch?

The Contestant

- rational agent playing with the goal of winning the prize

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The Host

- NOT a rational agent... playing with the same strategy every time and not maximizing his utility.

The Contestant

- rational agent playing with the goal of winning the prize

The Host

- a rational agent playing with a strategy to produce the most popular game show

Playing Fair

The fair host always reveals a goat and allows contestants to switch.

- switching $\implies P(win) = \frac{2}{3}$

Playing Evil

The evil host only allows contestants to switch if they initially pick the prize.

- switching $\implies P(win) = 0$

The Evil Showmaster

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Define Parameters

- p = the frequency with which the host plays evil
- q = the frequency with which the contestant will NOT switch when given the choice

Mind Reading Host

The host reaches his highest utility when he can accurately predict your behavior and adjust his strategy accordingly.

- high q or initially picked prize $\implies$ *fair*

Lying Contestant

The contestant can try to trick the host by pretending to have a high q and then switching doors at the last moment.

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- $P(\text{win}) = \frac{q}{3} \leq \frac{1}{3}$
- $P(\text{open}) = P(\text{stay}) + P(\text{switch})P(\text{car}) = \frac{1+2q}{3}$

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Credit Where Credit is Due

Link to References: https://docs.google.com/document/d/1NJ0qgb8TZdd31jMW8tWeP5G_wbnIGd1XaCHsh8Dhbzc/edit?usp=sharing