Problem 1. Let $E \subset [0, 1]$ be a measurable set such that there exists $\varepsilon > 0$ such that
\[
m(E \cap [a, b]) \geq \varepsilon |b - a|
\]
for all $[a, b] \subset [0, 1]$. Prove that $mE = 1$.

Problem 2. Let $E$ be a measurable set on the line and $f(x)$ a nonnegative measurable function on $E$ such that the limit,
\[
L = \lim_{n \to \infty} \int_{E} [f(x)]^n dx,
\]
exists, where $0 < L < \infty$. Prove that $m\{x \in E \mid f(x) = 1\} = L$.

Problem 3. Let $f$ be an integrable function on a set $E$ of a finite Lebesgue measure $mE < \infty$. Let $E_n = \{x \in E \mid |f(x)| \geq n\}$. Prove that
\[
\sum_{n=0}^{\infty} mE_n \geq \int_{E} |f(x)| \, dx \geq \sum_{n=1}^{\infty} mE_n.
\]

Problem 4. Prove that if $f$ is absolutely continuous on $[a, b]$ and $g(x) = e^{f(x)}$ then $g$ is absolutely continuous on $[a, b]$ as well.

Problem 5. Prove that if $f \in L^1[0, \infty)$, then
\[
\lim_{n \to \infty} \frac{1}{n} \int_{0}^{n} xf(x) \, dx = 0.
\]

Problem 6. Prove that if $f(x) \in L^{4+\varepsilon}[0, 1]$, where $\varepsilon > 0$, and $g(x) = f(x^2)$, then $g(x) \in L^2[0, 1]$. 