Problem 1. Let $E$ be a set on the line of a finite outer measure, $m^*E < \infty$, and
\[ S = \sup_{\text{measurable } A, A \subset E} mA. \]
Prove that $S = m^*E$ if and only if $E$ is measurable.

Problem 2. Prove that if $f(x)$ is an absolutely continuous function on the segment $[0,1]$ such that $f'(x) = 1$ almost everywhere, then $f(x) = x + C$, where $C$ is a constant.

Problem 3. Let $\{f_n(x), n = 1, 2, \ldots\}$ be a sequence of integrable functions on the segment $[0,1]$ such that
\[ \sum_{n=1}^{\infty} \|f_n\|_{L^1[0,1]} < \infty. \]
Prove that the series
\[ \sum_{n=1}^{\infty} \sin f_n(x) \]
converges in $L^1[0,1]$.

Problem 4. Let $F(f)$ be a bounded linear functional on $L^1[0,1]$ such that $F(f) = 0$ for all $f$ such that
\[ \int_0^1 f(x)dx = 0. \]
Prove that then there is a real number $C$ such that for all $f \in L^1[0,1],$
\[ F(f) = C \int_0^1 f(x)dx. \]

Problem 5. Prove that if $f_n$ converges to $f$ in $L^3[0,1]$, then $(f_n)^2$ converges to $f^2$ in $L^1[0,1]$.

Problem 6. Let $f$ be an integrable function on the segment $[0,1]$ such that for any $0 \leq a < b \leq 1,$
\[ \left| \int_a^b f(x)dx \right| \leq (b-a)^2. \]
Prove that $f = 0$ almost everywhere.