1. Describe and sketch the set $\text{Re}(\tan z) > 0$.

2. The region $D$ is the intersection of the disks $|z - 1| < \sqrt{2}$ and $|z + 1| < \sqrt{2}$. Construct a conformal mapping (one-to-one) from $D$ to the unit disk $|z| < 1$. First represent the mapping as a composition of elementary mappings, and then simplify the final formula as much as possible.

3. Let $f(z)$ be an entire function that does not take negative real values. Show that $f(z)$ is a constant function.

4. The function $f(z)$ is analytic in the unit disk $|z| < 1$ and maps it surjectively to the upper half plane $\text{Re} \, z > 0$. Find the radius of convergence of the Taylor series $\sum_{k=0}^{\infty} a_k z^k$ of the function $f(z)$.

5. a) How many solutions does the equation $2z^2 = \cos z$ have in the unit disk $|z| < 1$? b) Show that all solutions of the equation $2z^2 = \cos z$ that lie in the unit disk $|z| < 1$ are real.

6. Evaluate $\int_{0}^{\infty} \frac{\sin(\pi x)}{x - x^3} \, dx$. Explain each step.