Math 261 Practice Test 2 (11/2/09)
Ch. 14.6-14.8, 15.1-15.5

1. Find the maximum rate of change of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point (3, 6, –2) and the direction in which it occurs.

2. Find the equation of the tangent plane and the normal line to the surface $x - z = 4\arctan(yz)$ at the point $(1 + \pi, 1, 1)$.

3. Find the points on the hyperboloid $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

4. Find the local maximum and minimum values and saddle points of the function $f(x, y) = x^3 - 6xy + 8y^3$. Use the second derivative test to identify which points are which.

5. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x^2 y$ subject to the constraint $x^2 + y^2 = 1$.

6. A package in the shape of a rectangular box can be mailed by the US Postal Service if the sum of its length and its girth (the perimeter of a cross-section perpendicular to the length) is at most 108 in. Find the package with the largest volume that can be mailed.

7. Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin(y)$ and the planes $x = \pm 1$, $y = 0$, $y = \pi$, and $z = 0$.

8. Evaluate the integral by reversing the order of integration:

$$\int_0^1 \int_{\arcsin(y)}^{\pi/2} \cos(x)\sqrt{1 + \cos^2 x}\, dx\, dy\,.$$

9. In evaluating a double integral over a region D, a sum of iterated integrals was obtained as follows:

$$\iint_D f(x,y)\, dA = \int_0^1 \int_0^{2y} f(x,y)\, dx\, dy + \int_1^3 \int_0^{3-y} f(x,y)\, dx\, dy\,.$$

(con'd on other side)

(**9** Con'd)
Sketch the region D and express the integral as an iterated integral with reversed order of integration.

10. Find the volume of the solid bounded by the planes $z = x$, $y = x$, $x + y = 2$, and $z = 0$.

11. Use polar coordinates to find the volume of the solid which lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 1$.

12. Consider a lamina that occupies the region D beounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant with density function $\rho(x, y) = y$.

(a) Find the mass of the lamina.

(b) Find the center of mass.

(c) Find the moment of inertia of the lamina about the x-axis. What is the radius of gyration about the x-axis?