

MATH 22200

Sample FINAL EXAM Problems

*SHOW YOUR WORK***Problem 1** Find the derivative of the given function

a. $y = \text{Arcsin}(\ln 2x)$

ANSWER:

b. $y = e^{x^3} \cos(2x)$

ANSWER:

Problem 2 Evaluate

$$\lim_{x \rightarrow 0} \frac{x + \sin 2x}{x - \sin 2x}$$

ANSWER

Problem 3 Find the minima and maxima and points of inflection of the function $y = e^{-x} \sin x$, $0 \leq x \leq 2\pi$; sketch the graph.

Problem 4 Using standard forms, integrate:

$$(a) \int \frac{4x + 6x^2}{x^2 + x^3 + 1} dx$$

ANSWER:

$$(b) \int x e^{-x^2+3} dx$$

ANSWER:

$$(c) \int \frac{e^{-x} dx}{\sqrt{1 - e^{-x}}}$$

ANSWER:

$$(d) \int_1^3 \frac{dx}{x^2 - 2x + 5}$$

ANSWER:

Problem 5 Using standard forms and basic trigonometric identities integrate:

$$(a) \int \cos^2 2x \sin^3 2x dx$$

ANSWER:

$$(b) \int \sin^4 \frac{x}{2} dx$$

ANSWER:

Problem 6 Using integration by parts or substitution or integration by partial fractions, integrate:

$$(a) \int x \sin 2x dx$$

ANSWER:

$$(b) \int_3^5 \frac{\sqrt{x^2 - 9}}{x} dx$$

ANSWER:

$$(c) \int \frac{7 - x}{x^2 + x - 2} dx$$

ANSWER:

Problem 7 Find the volume of the solid of revolution obtained by rotating the first- quadrant area bounded by $y = x\sqrt[3]{x-1}$ and $x = 9$ about the y-axis.

ANSWER

Problem 8 Test the given geometric series for convergence or divergence. Find the sum of the series that is convergent.

$$16 + 12 + 9 + 27/4 + \dots$$

ANSWER

Problem 9 Find the first three nonzero terms of the Maclaurin expansion of the given function

$$f(x) = \cos 3x$$

ANSWER

Problem 10 Find the first three nonzero terms of the Maclaurin expansion of the given function by using the STANDARD expansions and operations with series:

$$f(x) = \ln(1 + x^2)$$

ANSWER

Problem 11 Find the first three nonzero terms of the Taylor expansion of the given function

$$\tan x, \quad a = \pi/4$$

Problem 12 Set up (but do not compute) the first three nonzero terms of the appropriate Taylor series for evaluation of $\tan 44^\circ$.

ANSWER

Problem 13 Find the first three nonzero terms of the Fourier series for the given function:

$$f(t) = \begin{cases} 0 & -\pi \leq t < 0 \\ 2t & 0 \leq t < \pi \end{cases}$$

Problem 14 Using separation of variables find the general solution the given differential equation.

$$2y(x^3 + 1)dy + 3x^2(y^2 - 1)dx = 0$$

ANSWER

Problem 15 Find the indicated particular solution of the given linear differential equation.

$$\frac{dy}{dx} - 4y = 2$$
$$y = 2 \quad \text{when} \quad x = 0$$

ANSWER

Problem 16 Solve the given differential equation.

$$2D^2y + Dy - 3y = 0$$

ANSWER

Problem 17 Find the particular solution of the differential equation for the given conditions (Use the method of undetermined coefficients).

$$D^2y + 4Dy + 4y = 4 \cos x$$

$$Dy = 1 \quad \text{and} \quad y = 0 \quad \text{when} \quad x = 0$$

ANSWER

Problem 18 Find the particular solution of the differential equation for the given conditions (Use Laplace transforms).

$$y'' + 4y = \sin 2t, \quad y(0) = 0, \quad y'(0) = 0$$

ANSWER