Differential Equations

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Objective

In this project we investigate the solutions of some differential equations.

Narrative

In this project we introduce the command `dsolve`:

|	
`dsolve(´Dy=-2*y´)`	solves the first-order equation $y' = -2y$.
`dsolve(´Dy=-2*y´,´y(0)=3´)`	solves the first-order equation $y' = -2y, y(0) = 3$.
`dsolve(´D2y+4*Dy+4*y=0´,´y(0)=1´,´Dy(0)=0´)`	solves the second-order equation $y'' + 4y' + 4y = 0$, $y(0) = 1, y'(0) = 0$.

Tasks

1. Type the commands below into MATLAB in the order in which they are listed. These commands solve some simple first order differential equations. (Is MATLAB's output correct? You should be able to solve these equations easily!)

   ```
   >> % Your name, today´s date
   >> % Differential Equations
   >> clear all, close all
   >> % Task 1
   >> y = dsolve(´Dy=-2*y´)
   ```
   Find the general solution to the first-order equation $y' = -2y$.

   ```
   >> y = dsolve(´Dy=-2*y´,´y(0)=3´)
   ```
   Find the specific solution to the first-order equation $y' = -2y, y(0) = 3$.

   ```
   >> figure(1), ezplot(y,[-1.0,1.0])
   ```
   Graph the last solution in the Figure 1 window.

2. Continue by typing the commands below into MATLAB in the order in which they are listed. The function $x_1 = x_1(t)$ denotes the solution to the differential equation $x'' + 4x = 0, x(0) = 1, x'(0) = 0$. This is the equation governing the motion of a spring-mass system consisting of a spring whose spring constant $k = 4$, whose mass is a unit mass $m = 1$, with no damping, and which is released after being stretched $x(0) = 1$ unit out of equilibrium (a simple harmonic motion).

   Observe that we are changing notation here: instead of writing $y = y(x)$ and a differential equation for y in terms of x, we're writing $x = x(t)$ and a differential equation for x in terms of t. We're changing the independent variable from x to t since we're interpreting the independent variable as time. And we're changing the dependent variable from y to x to emphasize the fact that when a spring-mass system is drawn in the position illustrated to the right, *the positive displacement from equilibrium is downward, not upward.* (And we use the command `set(gca,´ydir´,´reverse´)` to make sure that the direction of the positive x-axis in the Figure 2 window is correct.)
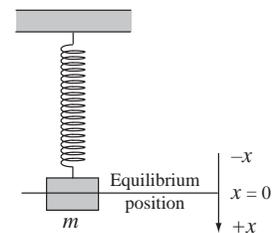
   Note: Pay careful attention to the shape of the curves drawn by each `ezplot` command in this task. At the end of this task the Figure 2 window will contain the graphs of 4 functions, and at the end of this project you will be asked to label each graph. To simplify labeling each graph, you might, in fact, find it useful to make a preliminary hard copy of the Figure 2 window after drawing each graph.

```
>> % Task 2
>> x1 = dsolve(´D2x+4*x=0´,´x(0)=1.0´,´Dx(0)=0.0´)
>> figure(2)
>> hold on, ezplot(x1,[0,2*pi]), set(gca,´ydir´,´reverse´), hold off
>> x2 = dsolve(´D2x+Dx+4*x=0´,´x(0)=1.0´,´Dx(0)=0.0´)
>> hold on, ezplot(x2,[0,2*pi]), set(gca,´ylim´,[-0.2,1.0]), hold off
>> x3 = dsolve(´D2x+5*Dx+4*x=0´,´x(0)=1.0´,´Dx(0)=0.0´)
>> hold on, ezplot(x3,[0,2*pi]), set(gca,´ylim´,[-0.2,1.0]), hold off

>> x4 = dsolve(´D2x+4*Dx+4*x=0´,´x(0)=1.0´,´Dx(0)=0.0´)
>> hold on, ezplot(x4,[0,2*pi]), set(gca,´ylim´,[-0.2,1.0]), hold off
```

The function $x_2 = x_2(t)$ denotes the solution to the differential equation $x'' + x' + 4x = 0, x(0) = 1, x'(0) = 0$. This is the equation governing the motion of the above spring-mass system, but now with a small amount of damping (underdamping). (The purpose of the `set(gca,´ylim´,[-0.2,1.0])` command is to restrict the range over which x_2 is graphed.) The function $x_3 = x_3(t)$ denotes the solution to the differential equation $x'' + 5x' + 4x = 0, x(0) = 1, x'(0) = 0$. This is the equation governing the motion of the above spring-mass system, but now with a large amount of damping (overdamping). And the function $x_4 = x_4(t)$ denotes the solution to the differential equation $x'' + 4x' + 4x = 0, x(0) = 1, x'(0) = 0$. This is the equation governing the motion of the above spring-mass system, but now with a critical amount of damping. (Any less damping results in underdamping, and any more damping results in overdamping.)

3. Continue by typing the commands below into MATLAB in the order in which they are listed. The purpose of these commands is to illustrate the effect of a forcing function $f(t)$ in a differential equation

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = f(t).$$

We begin by redrawing the graph of x_4 from Task 2 in Figure 3, and then drawing the graph of the solution $x_5 = x_5(t)$ to the differential equation $x'' + 4x' + 4x = 2\sin 4t, x(0) = 1, x'(0) = 0$.

```
>> % Task 3
>> figure(3)
>> hold on, ezplot(x4,[0,2*pi]), set(gca,´ydir´,´reverse´), hold off
>> x5 = dsolve(´D2x+4*Dx+4*x=2*sin(4*t)´,´x(0)=1.0´,´Dx(0)=0.0´)
>> hold on, ezplot(x5,[0,2*pi]), set(gca,´ylim´,[-0.2,1.0]), hold off
```

At this time, make a hard copy of MATLAB's Command Window and hard copies of its Figure 1, 2, and 3 windows. If you made any typing errors, neatly draw a line through them and any resulting MATLAB output, by hand. Then:

3. On the hard copy of your Figure 1 window, draw and label the t- and x-coordinate axes (be careful!).

4. On the hard copy of your Figure 2 window, draw and label the t- and x-coordinate axes (be careful!). Then label the graph of x_1 by "simple harmonic motion", the graph of x_2 by "underdamped motion", the graph of x_3 by "overdamped motion", and the graph of x_4 by "critically damped motion". (Only work on the final hard copy of the Figure 2 window: any preliminary hard copies of the Figure 2 window are for your reference only, and need not be supplemented by hand or turned in.)

5. On the hard copy of your Figure 3 window, draw and label the t- and x-coordinate axes (be careful!). Then label the graph of x_4 by "no forcing" and the graph of x_5 by "forced motion".

Your lab report for this project will be your hard copies of the Command Window and the Figure 1, 2, and 3 windows.