Radioactive Decay

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Objective

To study the decay of a radioactive material.

Narrative

Certain materials decay (they lose electrons, and hence mass) as time passes; these materials are known as *radioactive materials*, and the process of losing mass over time is known as *radioactive decay*. The number of grams of a radioactive material present at time t may be described by a function of the form

$$m(t) = m_0 e^{-kt} \tag{1}$$

where m_0 is the mass at time $t = 0$ and k is a some positive constant.

If, for example, $k = \ln 2$ then

$$m(t) = m_0 e^{-(\ln 2)t} = m_0 \left(e^{\ln 2}\right)^{-t} = m_0 \left(2\right)^{-t} = m_0 \left(\frac{1}{2}\right)^t . \tag{2}$$

Observe that $m(0) = m_0$, so m_0 is the mass of the material at time $t = 0$. And $m(1) = m_0/2$ so that when $t = 1$ there is half as much of the material left as there was at time $t = 0$. In general, there is half as much of the material left at time $t + 1$ as there was at time t in this case since

$$m(t+1) = m_0 \left(\frac{1}{2}\right)^{t+1} = m_0 \left(\frac{1}{2}\right)^t \frac{1}{2} = \frac{1}{2} m(t).$$

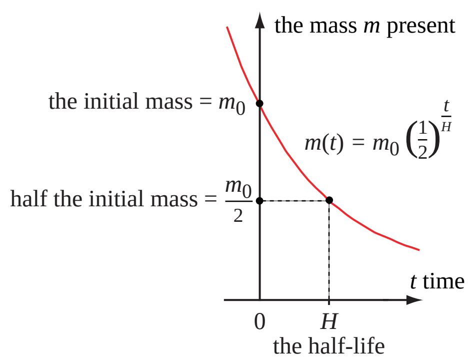
A more general version of (2) is the function

$$m(t) = m_0 \left(\frac{1}{2}\right)^{t/H}$$

where H is a constant known as the *half-life* of the material. Again, $m(0) = m_0$ so m_0 is the mass of the material present at time $t = 0$. But now it is when $t = H$ that there is half as much of the material left as there was when $t = 0$, for

$$m(H) = m_0 \left(\frac{1}{2}\right)^{H/H} = \frac{1}{2} m_0.$$

(Hence the term "half-life".) This is illustrated in the figure to the right.



Tasks

1. Type the command lines below into MATLAB. These commands produce a graph of $m(t) = 10e^{-0.1254t}$.

```
>> % Your name, today´s date
>> % Radioactive Decay
>> clear all, close all
>> m = inline(´10*exp(-0.1254*t)´)
>> ezplot(m,[0.0,10.0,0.0,10.0])
```

At this time, make a hard copies of MATLAB's Command Window and its Figure 1 window. If you made any typing errors, neatly draw a line through them and any resulting MATLAB output, by hand. Then:

2. On the graphic you created in Task 1:
   (a) estimate the half-life of the material whose mass is given by $m(t) = 10e^{-0.1254t}$, by drawing lines and labeling intercepts as in the figure above, by hand,
   (b) write a statement of the form, "The half-life is ____." by hand, filling in the blank with your estimate H of the half-life, and
   (c) check your estimate H of the half-life by computing $m(H)$ to 6 decimal places of accuracy by hand, and writing a statement of the form "$m(\_\_\_\_) = \_\_\_\_$." filling in the blanks with the appropriate values of H and $m(H)$.

Your lab report will be a hard copy of your typed input and MATLAB's responses (both text and hand-labeled graphics).

Comments

To find the half-life H of a radioactive substance given that m_0 grams are initially present and that $m(t) = m_0 e^{-kt}$ is present at time t, we need to find the time H at which

$$m(H) = m_0 e^{-kH} = \frac{1}{2} m_0. \tag{3}$$

We solve this equation by dividing by m_0:

$$e^{-kH} = \frac{1}{2} \tag{4}$$

taking the ln of each side:

$$-kH = \ln \frac{1}{2}$$

and dividing by $-k$:

$$H = \frac{\ln \frac{1}{2}}{-k} = \frac{\ln (2^{-1})}{-k} = \frac{-\ln 2}{-k} = \frac{\ln 2}{k}.$$

1. Does the answer you obtained graphically in this project agree with what you would have obtained had you tried to find the half-life algebraically?
2. Can you see how, in going from equation (3) to equation (4), we have eliminated m_0 from the problem? The significance of this is that *the half-life of a radioactive substance is independent of the amount of the substance initially present.*
3. Can you see how you could determine the half-life H of a radioactive substance given that m_1 grams are present at time $t = t_1$ and that m_2 grams are present at time $t = t_2$?