Maclaurin Series

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Objective

In this project we investigate the approximation of a function by its Maclaurin series.

Narrative

The Maclaurin series expansion of a function $f(x)$ is given by

$$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + \ldots,$$

and the functions:

$$f_0(x) = f(0)$$
$$f_1(x) = f(0) + f'(0)x$$
$$f_2(x) = f(0) + f'(0)x + \tfrac{f''(0)}{2!}x^2$$
$$f_3(x) = f(0) + f'(0)x + \tfrac{f''(0)}{2!}x^2 + \tfrac{f'''(0)}{3!}x^3$$
$$\vdots$$

form a sequence of better and better approximations to $f(x)$ by polynomial functions. In this project we illustrate this in the case of $f(x) = \sin x$. (Recall that the Maclaurin series expansion of $\sin x$ is

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots$$

where $n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$.)

Tasks

1. Type the commands below into MATLAB. These commands graph several Maclaurin series approximations to $f(x) = \sin x$. Check out the results of what you type in each figure window as it appears: you should observe that the greater `n`, the larger the interval over which `fn` is a good approximation of `f = sin(x)`.

```
>> % Your name, today´s date
>> % Maclaurin Series
>> clear all, close all
>> syms x
>> f = sin(x)
>> figure(1), ezplot(f,[-pi,pi])
>> f1 = x
>> figure(2), hold on, ezplot(f,[-pi,pi]), ezplot(f1,[-pi,pi]), hold off
>> f3 = x-x^3/6
>> figure(3), hold on, ezplot(f,[-pi,pi]), ezplot(f3,[-pi,pi]), hold off
>> f5 = x-x^3/6+x^5/120
>> figure(4), hold on, ezplot(f,[-pi,pi]), ezplot(f5,[-pi,pi]), hold off
>> f7 = x-x^3/6+x^5/120-x^7/5040
>> figure(5), hold on, ezplot(f,[-pi,pi]), ezplot(f7,[-pi,pi]), hold off
```

2. Continue by typing the following commands into MATLAB. These commands create a composite graphic.

```
>> figure(6)
>> hold on
>> ezplot(f,[-pi,pi])
>> ezplot(f1,[-pi,pi])
>> ezplot(f3,[-pi,pi])
>> ezplot(f5,[-pi,pi])
>> ezplot(f7,[-pi,pi])
>> hold off
```

At this time, make hard copies of MATLAB's Command Window and its Figure 6 window. (You may make copies of MATLAB's Figure 1–5 windows if you like, but you will not be asked to turn them in as part of your lab report, so it is not absolutely necessary to make copies of them.) If you made any typing errors, neatly draw a line through them and any resulting MATLAB output, by hand. Then, on your hard copy of the Figure 6 window:

3. using a straightedge, carefully draw the coordinate axes, and label them by hand,
4. highlight by hand in 4 different colors the graphs of $f1$, $f3$, $f5$, and $f7$, and
5. label by hand the graphs of f and $f1$, $f3$, $f5$, and $f7$. (Label the graph of f by "$\sin x$", the graph of of $f1$ by "$x - \frac{x^3}{3!}$", etc.)

Your lab report for this project will be your hard copies of the Command Window and the Figure 6 window.

Comments

The significance of being able to approximate a transcendental function such as $\sin x$, $\cos x$, e^x, or $\ln(1+x)$, by a polynomial is that polynomials are easier to work with (they're easier to evaluate) than transcendental functions. (If $f_7(x) = x - x^3/6 + x^5/120 - x^7/5040$, for example, then computing $f_7(2.0)$ can be done using the basic 4 operations of arithmetic — addition, subtraction, multiplication, and division — that are programmed into virtually all computer chips; computing sin(2.0), however, cannot be done so easily!).