# Newton's Method

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### *Objective*

In this project we study Newton's Method.

### *Narrative*

It is often necessary to solve equations in science, engineering and technology. Newton's Method is a quick and easy method for solving equations that works when other methods do not.

Newton's Method is actually a method for finding an arbitrarily good approximation to a value of $x$ for which $f(x) = 0$ (what is known as a *zero* of $f$). Newton's Method starts with finding an initial rough estimate $x_1$ to $x$, and then using the iterative equation

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \dots,$$

to obtain successively better and better approximations to $x$.

### *Task*

1. We begin by solving the equation $x^3 - 3x - 4 = 0$.

   (a) Type the commands below into MATLAB. These commands draw the graph of $f(x) = x^3 - 3x - 4$ over the interval $[-2, 3]$. We use this graph to make an initial rough estimate $x_1$ to the zero of $f$.

   ```
   >> % Your name, today´s date
   >> % Newton´s Method
   >> clear all, close all
   >> format long
   >> syms x
   >> % Task 1:  Solving x^3-3x-4=0
   >> f = inline(´x^3-3*x-4´)
   >> figure(1), ezplot(f,[-2.0,3.0])
   ```

   (b) At this time, make a hard copy of MATLAB's Figure 1 window. Using a straightedge, very carefully draw the coordinate axes on the graph of $f$. Then, again very carefully, label the coordinate axes and transfer the $x$- and $y$-tick marks and the $x$- and $y$-tick mark labels to the coordinate axes. Observe that the graph of $f$ intersects the $x$-axis near $x = 2$, but that it's difficult to be much more precise than that. (This is a major issue when solving "real" problems since it is often important to have very accurate numerical solutions to equations.)

   (c) Continue by typing the commands below into MATLAB. These commands apply the iterative equation presented in the Narrative to finding an accurate value of $x$ for which $f(x) = x^3 - 3x - 4 = 0$ using the initial rough estimate $x_1 = 2$.

   ```
   >> f1 = inline(´3*x^2-3´)
   >> M(1,1) = 1; M(1,2) = 2.0;
   >> for n=2:10 M(n,1) = n; x = M(n-1,2); M(n,2) = x-f(x)/f1(x); end
   >> M
   ```

   The first column of `M` contains the iteration number, and the second contains the corresponding approximation to $x$.

2. Repeat all parts of Task 1 — with appropriate modifications (such as drawing in a Figure 2 window rather than the Figure 1 window) — for the equation $x^3 - 2 = 0$. (As a result of this, you should be arriving at an approximation to $\sqrt[3]{2}$.)

At this time, make a hard copy of MATLAB's Command Window and hard copies of its Figure 1 and Figure 2 windows. If you made any typing errors, neatly draw a line through them and any resulting MATLAB output, by hand. Then:

Observe that in both equations we've studied, more and more decimal places "stabilize" the more times Newton's Method is iterated: the more decimal places that stabilize, the closer we are to the correct solution.

3. On your hard copy of MATLAB's Figure 1 window write the sentence, "The solution to the equation $x^3 - 3x - 4 = 0$ is approximately $x =$ _______." filling in the blank with the best possible approximation you can make to the correct solution on the basis of the data you have created.

4. On your hard copy of MATLAB's Figure 2 window write the sentence, "The solution to the equation $x^3 - 2 = 0$ is approximately $x =$ _______." filling in the blank with the best possible approximation you can make to the correct solution on the basis of the data you have created.

Your lab report for this project will be your hard copies of the Command Window and the Figure 1 and Figure 2 windows.

### *Comments*

In some presentations of Newton's Method the first rough estimate of the solution is denoted by $x_0$ rather than $x_1$ as we have done here. The reason we used $x_1$ instead of $x_0$ is that we wanted subscript indices to correspond to entries of `M`, and subscript indices must be strictly greater than 0 in MATLAB. (In other words, an array cannot have an index of 0 in MATLAB.)