2017 IUPUI High School Math Contest Problems

Visit http://math.iupui.edu/community/math-contest for details on how to compete.

1) Given any arc on a parabola (the part between any two distinct points), use compass and straightedge to construct its focus.

2) Four people, A, B, C, and D, are known to tell the truth only once out of every three times they speak. Each of them does so randomly, independent of what anyone previously said. They all speak; first A, then B, C, and finally D, who says "C negated that B said that A lied"

What is the probability that A told the truth in the first place? (We assume that B made a claim about A's statement, and that C made a claim about B's statement.)

3) There are n balls of unit mass and of zero radius on a 1-dimensional table of length 1, each currently either motionless or moving in either direction at a velocity identical for all moving balls. Assuming all the collisions are elastic (preserve the momentum and kinetic energy), show that the system is periodic (i.e. there is some time in the future when each billiard ball will again be in its current location and moving at its current velocity).

 4) Find all roots, real and complex, of

$$x^8 - 7x^7 + 14x^6 - 14x^5 + 27x^4 - 14x^3 + 14x^2 - 7x + 1 = 0$$

and express them in simplest form.

5) Write an essay of 500 to 700 words (complete with references) on how mathematics and billiards are related.

Students must work on their own when solving Problems 1-5 (above). They cannot receive help from their friends, teachers, or the internet.

Please go to the next page for the team problem.

<u>TEAM PROBLEM</u>

How many billiard balls of radius 1 can be arranged in a box of dimensions 55x78x101? (Hint: first try to solve the problem for smaller boxes.)

Teams of up to 4 students can work together to solve the team problem. (Individuals are welcome to participate as a one-person team.) Please remember to list all student's names on the team cover sheet.

Thanks: to William Cross and Rodrigo Pérez for submitting questions.