Math & Billiards
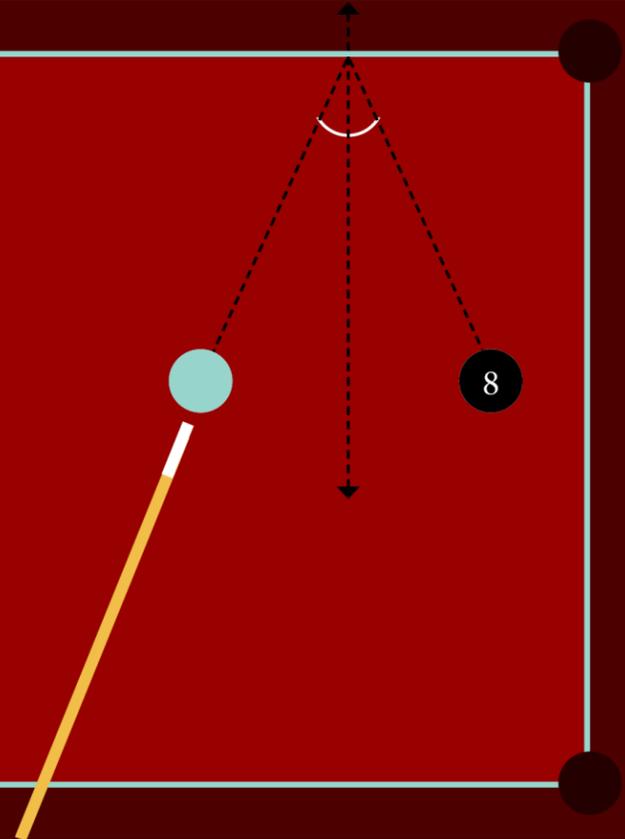## 2017 High School Math Contest
Presented by the IUPUI Department of Mathematical Sciences

IU IUPUI SCHOOL OF SCIENCE

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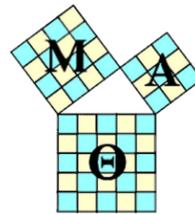
Mu Alpha Theta
National Mathematics
Honor Society for
High School Students

STUDENT PRIZES

One 1ˢᵗ place prize • $300 and a full 4-year tuition scholarship.*

Five 2ⁿᵈ place prizes • $150 each and a $2,000 scholarship.**

Ten 3ʳᵈ place prizes • $100 each and a $2,000 scholarship.**

Honorable mentions will receive a gift.

All entrants receive a certificate honoring their participation.

*In order to receive the scholarship, the winner must be directly admitted to the Purdue School of Science at IUPUI, major in any discipline in the School of Science, and attend full-time.

**Scholarships in the amount of $2,000 per year will be awarded to the remaining cash prizewinners who are directly admitted to the Purdue School of Science at IUPUI and attend full-time. These scholarships are renewable for four years, given satisfactory academic performance. Winners may receive only one scholarship from the School of Science, regardless of whether contest was won more than one year. Winners are also eligible for other campus-based scholarships, in addition to the one awarded through this contest.

RECENT MATHEMATICS DEPARTMENT AWARDS:

School Award:
2016 • Fishers High School
2015 • Carmel High School
2014 • Avon High School
2013 • Avon High School

Spirit Award:
2016 • Avon High School
2015 • Avon High School
2014 • MTI School of Knowledge
2013 • Fishers High School

ELIGIBILITY

This contest is open to all Indiana high school students (grades 9-12).

SUBMISSIONS

Submissions must be received by Friday, March 10, 2017, in order to be considered. Details are listed on the required cover sheets (used for both individual submissions and the team problem), which are posted on the website http://math.iupui.edu/community/math-contest. You may choose to solve one problem, several, or all five of the problems. Give your reasoning, not just the answers, and cite your sources and references appropriately. Entries will be judged by faculty in the IUPUI Department of Mathematical Sciences based on elegance of solutions as well as correctness.

CEREMONY

All participants will be invited to an awards ceremony at IUPUI on the afternoon of Friday, April 14, 2016. Parents and teachers will also be invited. The program will feature refreshments, a special presentation by Roland Roeder, Professor in the IUPUI Department of Mathematical Sciences, and will end with the award presentation.

Co Chairs: Roland Roeder and Maxim Yattselev

Contact Information:
http://math.iupui.edu/community/math-contest
IUPUI High School Mathematics Contest
Department of Mathematical Sciences
402 North Blackford Street, LD 270
Indianapolis, IN 46202-3216
(317) 274-6924 or contest@math.iupui.edu

PROBLEMS

1) Given any arc on a parabola (the part between any two distinct points), use compass and straightedge to construct its focus.

2) Four people, A, B, C, and D, are known to tell the truth only once out of every three times they speak. Each of them does so randomly, independent of what anyone previously said. They all speak; first A, then B, C, and finally D, who says "C negated that B said that A lied"

What is the probability that A told the truth in the first place? (We assume that B made a claim about A's statement, and that C made a claim about B's statement.)

3) There are n balls of unit mass and of zero radius on a 1-dimensional table of length 1, each currently either motionless or moving in either direction at a velocity identical for all moving balls. Assuming all the collisions are elastic (preserve the momentum and kinetic energy), show that the system is periodic (i.e. there is some time in the future when each billiard ball will again be in its current location and moving at its current velocity).

4) Find all roots, real and complex, of

$$x^8 - 7x^7 + 14x^6 - 14x^5 + 27x^4 - 14x^3 + 14x^2 - 7x + 1 = 0$$

and express them in simplest form.

5) Write an essay of 500 to 700 words (complete with references) on how mathematics and billiards are related.

Students must work on their own when solving Problems 1-5 (above). They cannot receive help from their friends, teachers, or the internet.

TEAM PROBLEM

How many billiard balls of radius 1 can be arranged in a box of dimensions 55x78x101? (Hint: first try to solve the problem for smaller boxes.)

Teams of up to 4 students can work together to solve the team problem. (Individuals are welcome to participate as a one-person team.) Please remember to list all student's names on the team cover sheet.

Thanks: to William Cross and Rodrigo Pérez for submitting questions.

(Photo courtesy of Andrzej Barabasz.)