

2016 IUPUI HIGH SCHOOL MATH CONTEST PROBLEMS

- 1) Martian tic-tac-toe is played on a 4×4 grid. The goal is to make a four in a row, but there is an extra move available: Instead of adding a symbol to the grid, you can choose to exchange all Xs for Os and all Os for Xs. This move is available only if your opponent did not swap immediately before you, and it is not available for the first move of the game. Show that the second player has a strategy to never lose (remember there can be ties).
- 2) Consider a convex polygon with 2016 sides (sides can have different lengths, but angles are all under 180 degrees). What is the largest possible number of obtuse angles? What is the largest possible number of acute angles?
- 3) For the 4th of July, a bowling alley will paint each of its bowling pins in either red, white, or blue. The pins are set up in the usual triangular configuration. How many different arrangements include all three colors, assuming that reflections and rotations of a given arrangement are considered the same?
- 4) Martian pool is played on a regular pentagonal table with one ball and no pockets. The ball always bounces off edges according to the equal angles rule, and is so small that it can be represented by a point. A "good shot" involves bouncing off two consecutive edges, but depending on the initial ball position, some good shots may not be possible.
 - a) Show that some good shot is possible from any initial ball position.
 - b) Find the region R of initial positions from where a good shot is possible on any pair of consecutive edges.
 - c) Find an initial ball position within the region R , and a direction that results in a "very good shot;" i.e., a shot in which the ball bounces off all five edges in consecutive order.
- 5) Write an essay of 500 to 700 words (complete with references) on an application of mathematics to study Mars.

Students must work on their own when solving Problems 1-5 (above). They cannot receive help from their friends, teachers, or the internet.

TEAM PROBLEM

The point with coordinates $(14,20)$ is "hidden" because it cannot be seen from the origin $(0,0)$ (the point $(7,10)$ blocks the view). The square formed by $(14,20)$, $(14,21)$, $(15,20)$, and $(15,21)$ is "hidden" because all its four points are hidden.

- a) Find the 3×3 hidden square (all its 9 points are hidden) that is closest to the origin.
- b) Now turn to 3-dimensional integer coordinates (a,b,c) , and find the $2 \times 2 \times 2$ hidden cube (all its 8 points are hidden) that is closest to the origin.

Teams of 2-4 students can work together to solve the team problem. (Please remember to list all student's names on the team cover sheet.)

Submissions must be received by Friday, March 18, 2016, in order to be considered. Details are listed on the required cover sheets (used for both individual submissions and the team problem), which are posted on the website <http://math.iupui.edu/community/math-contest>.