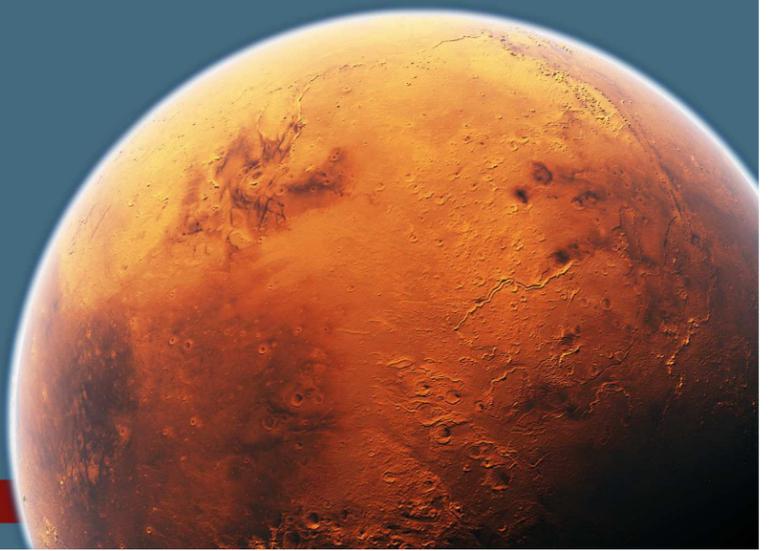


# Math and Mars

## 2016 High School Math Contest

Presented by the IUPUI Department of Mathematical Sciences

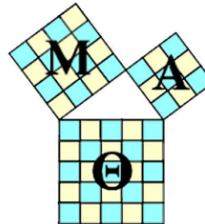
science@iupui



### 2016 Sponsors:



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Mu Alpha Theta  
National Mathematics  
Honor Society for  
High School Students

### STUDENT PRIZES

One 1<sup>st</sup> place prize • \$300 and a full 4-year tuition scholarship\*

Five 2<sup>nd</sup> place prizes • \$150 each and a \$2,000 scholarship\*\*

Ten 3<sup>rd</sup> place prizes • \$100 each and a \$2,000 scholarship\*\*

\*A full four-year academic tuition scholarship will be awarded to the first place prizewinner. In order to receive the scholarship, the winner must be directly admitted to the Purdue School of Science at IUPUI, major in any discipline in the School of Science, and attend full-time.

\*\*Scholarships in the amount of \$2,000 per year will be awarded to the remaining cash prize-winners who are directly admitted to the Purdue School of Science at IUPUI and attend full-time. These scholarships are renewable for four years, given satisfactory academic performance. Honorable mentions will receive a gift. All entrants will receive certificates honoring their participation.

### RECENT MATHEMATICS DEPARTMENT AWARDS:

#### School Award:

2015 • Carmel High School  
2014 • Avon High School  
2013 • Avon High School  
2012 • Carmel High School  
2011 • Brownsburg High School

#### Spirit Award:

2015 • Avon High School  
2014 • MTI School of Knowledge  
2013 • Fishers High School  
2012 • Avon High School

### ELIGIBILITY

This contest is open to all Indiana high school students (grades 9-12).

### SUBMISSIONS

Submissions must be received by Friday, March 18, 2016, in order to be considered. Details are listed on the required cover sheets (used for both individual submissions and the team problem), which are posted on the website <http://math.iupui.edu/community/math-contest>. You may choose to solve one problem, several, or all five of the problems. Give your reasoning, not just the answers, and cite your sources and references appropriately. Entries will be judged by faculty in the IUPUI Department of Mathematical Sciences based on elegance of solutions as well as correctness.

### CEREMONY

All participants will be invited to an awards ceremony at IUPUI on the afternoon of Friday, April 15, 2016. Parents and teachers will also be invited. The program will feature refreshments, a special presentation by Rodrigo Pérez, Professor in the IUPUI Department of Mathematical Sciences, and will end with the award presentation.

**Co Chairs:** Jeffrey Watt, Roland Roeder

**Thanks:** to William Cross and Rodrigo Pérez for submitting questions and to NASA for the photo of Mars and artists' rendition of the Mars Rover.

#### **Contact Information:**

<http://math.iupui.edu/community/math-contest>  
IUPUI High School Mathematics Contest  
Department of Mathematical Sciences  
402 North Blackford Street, LD 270  
Indianapolis, IN 46202-3216  
(317) 274-6924 or [contest@math.iupui.edu](mailto:contest@math.iupui.edu)

### PROBLEMS

- 1) Martian tic-tac-toe is played on a 4x4 grid. The goal is to make a four in a row, but there is an extra move available: Instead of adding a symbol to the grid, you can choose to exchange all Xs for Os and all Os for Xs. This move is available only if your opponent did not swap immediately before you, and it is not available for the first move of the game. Show that the second player has a strategy to never lose (remember there can be ties).
- 2) Consider a convex polygon with 2016 sides (sides can have different lengths, but angles are all under 180 degrees). What is the largest possible number of obtuse angles? What is the largest possible number of acute angles?
- 3) For the 4th of July, a bowling alley will paint each of its bowling pins in either red, white, or blue. The pins are set up in the usual triangular configuration. How many different arrangements include all three colors, assuming that reflections and rotations of a given arrangement are considered the same?
- 4) Martian pool is played on a regular pentagonal table with one ball and no pockets. The ball always bounces off edges according to the equal angles rule, and is so small that it can be represented by a point. A "good shot" involves bouncing off two consecutive edges, but depending on the initial ball position, some good shots may not be possible.
  - a) Show that some good shot is possible from any initial ball position.
  - b) Find the region R of initial positions from where a good shot is possible on any pair of consecutive edges.
  - c) Find an initial ball position within the region R, and a direction that results in a "very good shot;" i.e., a shot in which the ball bounces off all five edges in consecutive order.
- 5) Write an essay of 500 to 700 words (complete with references) on an application of mathematics to study Mars.

**Students must work on their own when solving Problems 1-5 (above). They cannot receive help from their friends, teachers, or the internet.**

### TEAM PROBLEM

The point with coordinates (14,20) is "hidden" because it cannot be seen from the origin (0,0) (the point (7,10) blocks the view). The square formed by (14,20), (14,21), (15,20), and (15,21) is "hidden" because all its four points are hidden.

- a) Find the 3x3 hidden square (all its 9 points are hidden) that is closest to the origin.
- b) Now turn to 3-dimensional integer coordinates (a,b,c), and find the 2x2x2 hidden cube (all its 8 points are hidden) that is closest to the origin.

**Teams of 2-4 students can work together to solve the team problem. (Please remember to list all student's names on the team cover sheet.)**

