

December 11, 2010

**MATH 166**

FINAL EXAM FALL 2010 (109 points)

**SHOW YOUR WORK PLEASE**

**Problem 1** (14 points) Find the derivative of  $f$

a)  $f(x) = \tan^{-1}(x) \ln 4x$

ANSWER:

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b)  $f(x) = e^{x^3} \ln x$

ANSWER:

**Problem 2** (7 points) Find an equation of the tangent line to the curve  $xe^y + ye^x$  at the point  $(0, 1)$ .

**Problem 3** (21 points) Evaluate

$$(a) \int \frac{x}{\sqrt{1-x^4}} dx$$

ANSWER:

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$$(b) \int \frac{\ln x dx}{x\sqrt{1+(\ln x)^2}}$$

ANSWER:

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$$(c) \int \frac{\sec x \tan x}{\sec^2 x - \sec x} dx$$

ANSWER:

**Problem 4** (8 points) Determine if the following integral is convergent or divergent and if it is convergent find its value:

$$\int_0^3 x^2 \ln x dx$$

ANSWER:

**Problem 5** (8 points) A sample of carbon-14 decreases in time at a rate that is proportional with the amount of sample present. The half life time of a sample is 5700 years.

Find the age of a sample in which 15 percent of the original radioactive nuclei originally present have decayed.

ANSWER:

**Problem 6** (7 points) Set up a numerical estimate of the following integral using the midpoint sum with  $n = 6$ :

$$\int_0^3 x^2 e^{x^2} dx$$

ANSWER:

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**Problem 7** (7 points) Find the area of the surface obtained by rotating about the  $x$  axis the following curve.

$$x = 3t - t^3, \quad y = 3t^2, \quad 0 \leq t \leq 1$$

ANSWER:

**Problem 8** (8 points) Sketch the polar curves, and set up (but do not evaluate) the integral that represents the area of the region that lies inside the first curve and outside the second curve:

$$r = 3 \cos \theta, \quad r = 2 - \cos \theta$$

ANSWER:

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**Problem 9** (7 points) Test the series for convergence or divergence

$$\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^5 + 4}$$

ANSWER:

**Problem 10** (7 points) Find the radius of convergence and the interval of convergence of the series. If the interval is finite do not forget to test the end points.

$$\sum_{n=1}^{\infty} \frac{n}{7^n} (x+6)^n$$

ANSWER:

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**Problem 11** (8 points) Find the Maclaurin series and its radius of convergence for  $f$

$$f(x) = \frac{x}{1+3x^2}$$

ANSWER:



**Bonus Problem** (7 points) Find the limit

$$\lim_{x \rightarrow \infty} (e^x + x)^{\frac{1}{x}}$$

ANSWER: