

NO CALCULATORS ALLOWED

December 8, 2007

MATH 164

FINAL EXAM FALL 2007 ( 100 points)

SHOW YOUR WORK PLEASE

**Problem 1** (14 points) Find the derivative of  $f$

a.  $f(x) = \sin^{-1}(\ln 2x)$

b.  $f(x) = e^{x^2} \log_{10} x$

**Problem 2** ( 9 points) Find the minima and maxima and points of inflection of the function  $y = xe^{-x}$ ; sketch the graph.

**Problem 3** (21 points) Evaluate

(a)  $\int \frac{1}{x \ln x} dx$

(b)  $\int \frac{e^x dx}{\sqrt{1 - e^{2x}}}$

(c)  $\int \frac{x}{\sqrt{x+1}} dx$

**Problem 4** (7 points) Evaluate

$$\int_0^{\infty} xe^{-x} dx$$

**Problem 5** (7 points) Find the volume obtained by rotating the region bounded by  $y = \cos x$ ,  $y = 0$ ,  $x = 0$  and  $x = \pi/2$  about the  $x$ -axis.

**Problem 6** (7 points) Solve the differential equation

$$y'x + y = y^2, \quad y(1) = -1$$

**Problem 7** (7 points) Find the length of the parametric curve.

$$x = 2 + \frac{1}{2}t^2, \quad y = 3 + \frac{1}{3}t^3, \quad 0 \leq t \leq 1$$

**Problem 8** (7 points) Sketch the polar curves and set up, but do not evaluate the integral that represents the area of the region that lies inside both curves.

$$r = 2 + \cos \theta, \quad r = 2 + \sin \theta$$

**Problem 9** (7 points) Test the series for convergence or divergence

$$\sum_{n=1}^{\infty} \frac{\arctan n}{n^3}$$

**Problem 10** ( 7 points) Find the radius of convergence and the interval of convergence of the series. If the interval is finite do not forget to test the end points.

$$\sum_{n=1}^{\infty} \frac{n}{5^n} (x+2)^n$$

**Problem 11** (7 points) Find the Maclaurin series for  $f$

$$f(x) = \frac{x^2}{1+x}$$

**Bonus Problem** ( 7 points) Find the limit

$$\lim_{x \rightarrow 0} \frac{1}{x^3} \int_0^x \sin^2 t dt$$