

Math 164 Final Exam, Spring 2008

1. (10 points) Differentiate f

(a)

$$f(x) = 10^{\sqrt{x}}$$

(b)

$$f(x) = \sin^{-1} e^{2x}$$

2. (10 points) Find an equation for the tangent line to the graph of $f(x) = x^2 \ln x$ at the point $(1, 0)$.

3. (10 points) Evaluate

$$\lim_{x \rightarrow \infty} (x \tan(1/x))$$

4. (18 points) Evaluate the integrals

(a)

$$\int \frac{\ln x}{\sqrt{x}} dx$$

(b)

$$\int \frac{\sqrt{x^2 - 1}}{x} dx$$

(c)

$$\int \frac{x^2 - x + 6}{x^3 + 3x} dx$$

5. (10 points) Find the length of the curve $y^2 = (x - 4)^3$ from the point $(5, 1)$ to the point $(8, 8)$.

6. (10 points) Set up **but do not evaluate** an integral that will give the area inside both the curves $r = 2$ and $r^2 = 4 \cos 2\theta$.

7. (10 points) Determine if the sequence $\{\tan^{-1} 3n\}$ converges or diverges. If it converges find the limit.

8. (12 points) Use a test to determine whether the series converges or diverges. State which test you are using.

(a)

$$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^3 + 1}}$$

(b)

$$\sum_{n=0}^{\infty} \frac{\sin(n\pi/2)}{n!}$$

9. (10 points) Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^2 3^n}.$$

Bonus. (8 points) Give a power series expansion for $f(x) = \tan^{-1} x$. Use this to express $\pi/4$ as an infinite sum.