

Math 164 Final Exam, Fall 2008

1. Find y' if

(a)

$$y = 2^x \ln 4x$$

(b)

$$y = \arctan e^{x^2}$$

2. Find

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x} + \frac{1}{x^2} \right)^x$$

3. Evaluate the following integrals.

(a)

$$\int x \ln x dx$$

(b)

$$\int \frac{x^2}{\sqrt{(x^2 + 4)^3}} dx$$

(c)

$$\int \frac{x^2 + 1}{x(x^2 + 4)} dx$$

4. Evaluate the integral.

$$\int_0^1 \frac{dx}{\sqrt{1-x}}$$

5. Find an equation of the line which is tangent to the parametric curve $x = 2 \cos t, y = \sin t$, with $0 \leq t \leq \frac{\pi}{2}$, at the point $(\sqrt{2}, \frac{\sqrt{2}}{2})$.

6. Find the length of the arc of the curve $y^2 = 2x^3$ from point $(0, 0)$ to point $(2, 4)$.

7. Set up, **but do not evaluate**, an integral which represents the area of the region which lies between the curves:

$$r = 1 + \sin \theta$$

$$r = 1 - \sin \theta$$

8. Determine the limit of the following sequence, if it exists. Make sure and show all work involved in the calculation.

$$\left\{ \frac{1 - 3n}{4n + 1} \right\}_{n=1}^{\infty}$$

9. Determine whether or not the following series is conditionally convergent, absolutely convergent, or divergent. State carefully which test you are using.

$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^2 + 1}$$

10. Determine whether or not the following series converges or diverges. State carefully which test you are using.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3 + n + 1}}$$

11. Find the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{n^2}{10^n} (x + 7)^{n+1}$$

12. Find the Maclaurin series for

$$\frac{2x}{1+x^2}$$

Bonus Determine whether or not the following series converges or diverges. If it converges, find the sum of the series.

$$\sum_{n=0}^{\infty} \frac{2^n}{n!}$$