Math 166 **Final Exam** **May 2, 2009**

Exam is 8 pages plus cover page. Follow the instructions for each question.
Show enough of your work that we can understand what you are doing.

(12 points) **1.** Find the derivatives of the following functions

(a) $f(x) = e^{x^2-5x+\sqrt{2}}$

(b) $g(x) = x^3 \tan^{-1} x$

(c) $h(x) = \ln\left(\dfrac{x}{\sin^{-1} x}\right)$

(8 points) **2.** Find the maximum and minimum values of $f(x) = xe^{-2x}$ on the interval $[0, 5]$

(6 points) **3.** Find $\lim_{x \to 2} \dfrac{\ln(x-1)}{x^2 - 4}$

4. Find

(6 points) (a) $\displaystyle\int \frac{x+2}{x^3+x}\,dx$

(6 points) (b) $\displaystyle\int \sqrt{1-x^2}\,dx$

(8 points) **5.** Evaluate

$$\int_1^4 \sqrt{x} \ln x \, dx$$

(6 points) **6. Set up** an integral to find the length of the parametric curve $x = t^2 + 2t$, $y = t^3 + t$, where $-1 \leq t \leq 2$. You do **not** need to evaluate this integral.

(8 points) **7.** Sketch the graph of the polar coordinate equation $r = 2 + \cos(3\theta)$, labeling a few points on the curve with the related values of θ.

Set up an integral to find the area inside the curve.
You do **not** need to evaluate the integral.

(8 points) **8.** Find $\lim_{n\to\infty} \dfrac{\sqrt{2n^2+1}}{2-5n}$ Show work!!

(8 points) **9.** Decide whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$ converges or diverges.

Say which test you are using to show convergence/divergence, show your work in carrying out the test, and state your conclusion.

(8 points) **10.** Decide if the series $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^3+2}$ is absolutely convergent, is conditionally convergent, or the series diverges. Say which test(s) you are using to show convergence/divergence, show your work in carrying out the test(s), and state your conclusion.

(8 points) **11.** Find the Maclaurin series for $f(x) = \frac{x}{3-x}$

(8 points) **12.** The function g is given by the Taylor series $g(x) = \sum_{k=0}^{\infty} \frac{(x-3)^k}{(k+1)2^k}$

Find the interval of convergence for the Taylor series.
(*Be sure to indicate which of the the end points, if any, are included in the interval!*)

(8 points) **Bonus** Find the length of the parametric curve $x(t) = \frac{1-t^2}{1+t^2}$ and $y(t) = \frac{2t}{1+t^2}$ for $-1 \le t \le 1$.